SINGLE VENDOR MULTI BUYERS SUPPLY CHAIN OPTIMIZATION

BY

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This piece of work is dedicated to my wonderful and always giving parents whose support and prayers led to this achievement.

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LIST OF ABBREVIATIONS

D:	Annual rate of demand for the manufacturer
P:	Annual rate of production for the manufacturer
h:	Inventory carrying cost per item per year for the manufacturer
S:	Production set up cost per lot
Z:	The large production batch size of manufacturer
T:	The cycle time of vendor production and shipments to all buyers
n:	Number of equal sized batches in a lot
m:	Total number of shipments sent to all buyers in a cycle T.
L:	Number of unequal shipments sent to any buyer
i:	Denote for the i the buyer i= (1, 2,...,m)
Di:	Annual rate of demand for each buyer $D=\sum_{i=1}^{m} D_i$
h_i:	Inventory carrying cost per item per year for each buyer
S_i:	Cost of placing an order for each buyer
T_i:	Cost of transporting a batch from the manufacturer to the buyer i
q_i:	The smallest batch size that has been sent to buyer i in one shipment

K: The ratio of production rate to the total demand

Q: The big lot size sent by vender in each cycle

Q_i: The part of the big lot size in each cycle that has been sent to buyer i only

ZSR: The zero switch rule.

NFSF : Near feasible solution finder.

ABSTRACT

Full Name : Illias Abdulkareem Musliyar

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This research is about the coordination between a single vendor and multiple buyers in supply chain optimization. It investigates the problems of single vendor multiple buyers by studying the strength and weaknesses of relevant state-of-the-art models in the literature. In this regards, we have succeeded in providing two extensions to recent studies. The first extension is in the methodology and the second extension is in relaxation of existing model.

Finally, we proposed two heuristic methods to investigate the generic problem of single vendor and multiple buyers. Experimental results suggest that our proposed methods provide better solution as compared to those available in literature.

ملخص الرسالة

الاسم الكامل: إلياس عبد الكريم موصلي يار

عنوان الرسالة: الحلول المثلى لخطوط الإمداد التي تتكون من مورد واحد وعدة مستهلكين.

التخصص: هندسة النظم الصناعية

تاريخ الدرجة العلمية: ماجستير علوم هندسة صناعية

في هذه الرسالة تمت دراسة أفضل النماذج الموجودة في العالم في الوضع الراهن وتمت دراسة نقاط القوة والضعف فيها. تم تطبيق بعض الامتدادات والتعديلات على هذه النماذج بحيث يكون لدينا أفضل الحلول التي تضمن أكثر الانتاجية وأقل التكاليف.

تم التعامل مع نماذج تعتمد على ارسال الشحنات لجميع المستهلكين في نفس الوقت واثبات أن تلك النماذج مبنية على فرضيات تضعف من هذه النماذج وتسبب في تكاليف أعلى خصوصا في حال تعدد المستهلكين. لقد تم تصميم نموذج أفضل من تلك النماذج بما يحقق أقل التكاليف وأكثر انتاجية إلى جانب كون الحلول سهلة ويسيرة.

وتم التعامل أيضا مع نموذج على الارسال الترددي المنتظم بين المستهلكين وتم الاثبات بانه نموذج يقوم على فرضية من شانها أن تسبب في عدم انخفاض التكاليف. تم تصميم نموذج أكثر مرونة من نموذجه وتم الاثبات عن طريق الأمثلة إلى أنها توفر نتائج أفضل وقد تم توفير 11% تقريبا من التكاليف في احدى الحالات.

أخيرا تمت دراسة أكثر النماذج المرنة في العالم حيث تم تصميم منهجيتي حل عشوائي للوصول إلى النتائج بأقل الجهد والتكاليف وأكثر الأرباح وتم اثبات أن الحل العشوائي الوارد في أفضل النماذج المرنة في عالم البحوث ليس حلا أمثل وهناك طرق حل عشوائية أفضل في حل هذه المسائل.

CHAPTER 1

INTRODUCTION

Many companies around the world realize that their internal efficiency is not sufficient enough to position them in today's global competitive market. It is obvious that nowadays, working alone cannot lead to a competitive business. So, in order to have a better customer service and lower supply chain cost, there is need to synchronize supply chain management network. A speedy response to sudden change in demand as well as delivery at the right time is difficult to achieve in the absence of collaboration along the supply chain.

The inventory management problem of finding the optimal shipments and deliveries between the vendors and buyers has been studied widely in literature. Studying the whole network of vendors, we notice that buyers construe various solutions and patterns of the relationship strategies. Various solutions to "single vendor - single buyer" and "single vendor-multi buyers" in the context of consignment stock have been discussed in literature. And various strategies such as "shipping to all buyers at the same time or at different times", "sending equal or unequal shipments" and "sending shipment right after the production or waiting until the last shipment is completely consumed" have been proposed in order to lower the cost of the supply chain network

In this work we focus on the recent models in the literature and investigate some of the assumptions that made the models least favorable and incurred larger costs in practice.

1.1 Background of the Study

Single vendor multi buyer is relationship between vendor and many buyers. In this area of study we look at the way that vendor and buyers are working, when vendor is producing, shipping and how much, when we deliver the shipments, and how buyer should deal with shipments. These arrangements lead us to study the cost incurred by these arrangements mainly on the cost of vendor inventory , buyer inventory, transport, ordering cost and setup cost. The costs are studied in a centralized cost strategy that we have one total cost of the whole system.

In order to come up with the most effective models that lead to better solutions, we investigated more than 12 state-of-the-art models.

1.2 Problem Definition

Using centralized cost strategy offers better solutions to “single vendor-multi buyer” supply chain systems. However, the state-of-the-art models despite that they offer better solutions were found to have weaknesses that made them least favorable and incurred larger costs in practice.

1.3 Research Objectives

I. To come up with much effective models that lead to better solutions to "single vendor-multi buyer" supply chain systems

II. To propose heuristic method to investigate the generic problem of "single vendor-multi buyer".

III. To compare the performance of our proposed methods with those available in literature.

1.4 Research Methodology

In this work we adopted a systematic approach, in which we focus on state-of-the-art models and their variants from literature. We provide detailed discussion of the models and our proposed extensions, make two type comparisons namely; theoretical comparison whenever possible and numerical comparison. And finally we present comparative studies of the model and our proposed extensions of the models.

1.5 Organization of the Thesis

The rest of this report is organized as follows. Chapter two present literature review of various recent models. In chapter 3 we focus on Hoque [16, 18] models and proposed improved extensions on them. While in chapter 4, we discussed Bendaya model [6] and relaxed certain restrictions on the model to come up with an improved extended model to the problem.

We also proposed heuristic methods for the most general model in literature in chapter 5. The proposed heuristic provided better solution than that proposed in [20]. Finally, chapter 6 provides a summary of the work, conclusion and proposed future studies.

CHAPTER 2

LITERATURE REVIEW

"Single vendor single buyer" approach was analyzed in 1986 by Banerjee in [3]. The integrated vendor – buyer model was analyzed as "lot for lot" model in which a vendor produces for each buyer in separate batch. Goyal in [10] illustrated that producing a batch which is made up of certain number of equal shipments will lead to a lower cost. However, Goyal in [11] considered another strategy to produce unequal shipments with an increasing factor of the ratio of "production rate" to "demand rate", this gives lower cost. Meanwhile, Hill in [14] have shown through numerical examples that the increasing factor of Goyal in [11] could be relaxed to be in the range of one up to the ratio of "production rate" to "demand rate" and gives better solutions. Furthermore, Goyal in [12] found an improvement to the approach for the optimal policy of the integrated inventory system considering the capacity constraint determined by the transport equipment. Zavanella in [8] discussed an industrial case and performance analysis of consignment, while Zanoni in [32] provided a full analytical solution for the case in Zavanella [8]. Ben-Daya and Hariga in [5] relaxed the assumption of deterministic demand and assumed that the lead-time was varying linearly with the lot size. They considered delay times and lot size dependent run time. Hoque in [15] introduces a heuristic solution to minimize the total cost of setup ordering, inventory holding and lead-time. Zhou and Wang in [31] extended a model, which requires the buyer's unit holding cost to be greater than the vendors holding cost.

For “single vendor multi buyer” systems, Lal and Staelin in [25] developed a quantity discount schedule for those vendors who faced many groups of same category of buyers however, their model had shortages. Joglekar in [22] observed that in multi-buyers situation vendor order sizes affected not only the vendor’s revenue but also his production cost. Joglekar and Tharthare in [23] found an individually responsible and rational decision method to the economic lot sizes for “one vendor multi buyer” case. They opposed the previous models and termed them as antithetical to free enterprise system. Banerjee in [4] developed a model for inventory control between a vendor and multi buyers, who deal with a single product and focused their attention on the use of Electronic Data Interchange (EDI). They argued that EDI would form a link between the parties and monitor the consumption pattern of the buyers. They assumed that the parties deal with a single product and agreed to ship the materials at fixed intervals. In a regular case, the shipment size by vendor depends on the quantity at hand as prearranged products replenish up to quantity. Lu in [26] commented that all previous studies assumed that the vendor must have known the buyers holding and ordering costs, which were difficult to evaluate unless the buyer was willing to clarify the true values. Lu changed the objective to be “to minimize” the vendor’s total cost per year subject to maximum cost that the buyer can be charged. Viswanathan in [28] gave a model to analyze the benefit of “supply chain inventory coordination” using common replenishment agreements for a single product but they did not include the cost of vendor inventory in the model.

Woo in [29] considered another integrated inventory management model and produced the product for multi buyers in the same cycle. They used the same model of Banerjee in

[4] and extended the model using EDI-based inventory control systems in order to decrease the joint cost. Boyaci and Gallego in [7] analyzed pricing policies with inventory management, which jointly maximized the channel profit in a supply chain. This consists of one wholesaler and one or more retailers under price sensitive customer demand. They illustrated how an optimal policy could be implemented with an inventory consignment based on an agreement for coordination. Kim in [24] and Abdul-Jalbar in [1] also considered the “single vendor multi-buyer” supply chain modeling. Kim in [24] proposed an analytical model in order to integrate the purchase of a raw material, the production of multiple items utilizing the raw material and their delivery to multiple buyers. The objective was to find the production sequences of items, the common production cycle length & the delivery frequencies and quantities that minimize the average total cost. In Abdul-Jalbar model, the demand of buyer for any item was assumed to be known and satisfied by the item in the store and the goal was to determine one-cycle policies that minimized the average total cost. However, these “single vendor multi buyer” models do not considered an integrated inventory when a single product was produced by the vendor and delivered to multiple buyers. Hoque in [16] proposed three different models on “single vendor multi buyers” synchronizing system. The main policy of his model was to distribute the yielded shipments among all buyers at the same time so that the quantity received by any buyer is proportional to his demand rate. Two of Hoque models were based on equal shipments but the problem was that the inventory accumulated in buyer or vendor’s store. The third model used the idea of Goyal in [11] where the vendor produced unequal shipments with a factor of the ratio of production rate to demand rate. Hoque in [17] introduced another idea of having a combination of equal

and unequal shipments as in [32]. Zavanella and Zanoni in [32] investigated the principles of a Vendor Managed Inventory (VMI) policy, which was known as Consignment Stock (CS). CS represented a successful strategy for both vendor and buyer. They proposed a new model where the vendor sent equal shipments to the buyer in a cyclical pattern i.e. the vendor sends one shipment to the first buyer and then another one to the next buyer and then returns to the same sequence in the same cycle. However, Hoque thought the sequence was not important and did not affect the model. The same Zavanella and Zanoni in [33] released a note where they modified their model to be in consecutive shipments instead of cyclic approach. But, Hoque in [18] returned with a more generalized idea of mixed shipments of equal and unequal sizes. He tested the model by two examples involving two buyers and five buyers respectively and then compared the results with the result in [16]. Ben-Daya et al in [6] modeled CS and VMI policy for "single vendor multi buyers" supply chain system with known demand. They treated three cases where the vendor and buyers act independently. The vendor entered in a VMI and CS, and integrated firm where vendor and buyers costs were centralized. Hariga et al in [20] relaxed any constraints on the type of vendor shipments in literature to have free vendor shipments with various sizes of batch, number of shipments and schedule of deliveries to any buyer. The problem in this theory was introduced as that of scheduling and lot sizing modeling, under consignment

stock and centralized supply chain management system. Hariga et' al provides a nonlinear mixed-integer programming formulation for the general scheduling and lot-sizing problem, which is an NP – hard problem as explained. Also, they proposed a heuristic procedure to generate a near- optimal delivery schedule. They showed that their model gave better cost savings that increased as the number of buyers increased, as compared to Zavanella in [33].

CHAPTER 3

EXTENSIONS OF SENDING AT THE SAME TIME POLICY

3.1 Hoque [16] Model I extension

This chapter considered the first model of Hoque in [16], where he assumed that the shipment of the vendor is to be sent just after production. This implies that, the inventory would accumulate with buyers rather than with vendors.

3.1.1 Hoque Model System Policy:

The model was developed based on the following assumptions:

- Deterministic constant demand and production rates for the whole system
- Each buyer estimated individual demand (d_i) , holding and ordering costs (h_i, s_i) under various cost factors and informed the manufacturer.
- The concerned parties shared the benefits of coordination based on negotiation in a costless way.
- There was no backlogging or deliberate planning for shortages.
- Both the manufacturer and the buyers had enough storage capacity to accommodate the required inventory.
- The transport equipment had enough capacity to transport any of the batches to a buyer.

- Set-up and transportation times were insignificant.
- N equal batches of size Z each are produced in each production run.
- The buyers would pay the ordering cost once in each cycle but he pays for the transportation cost for each batch zi.

Note that we had used the same notations that we introduced in the first chapter.

3.1.2 Hoque [16] Model I

Suppose the manufacturer transferred the batch *z* to meet the demands of all of the buyers in different amounts of z_i and since the period of time for each batch consumption is given as: $\frac{z_i}{D_i} = z/D$

$$z_i = D_i * z/D \tag{3.1}$$

So that $$z = \sum_{i=1}^{m} z_i . \tag{3.2}$$

Figure 3.1 shows the clear picture of the inventory level for both manufacturer and buyer.

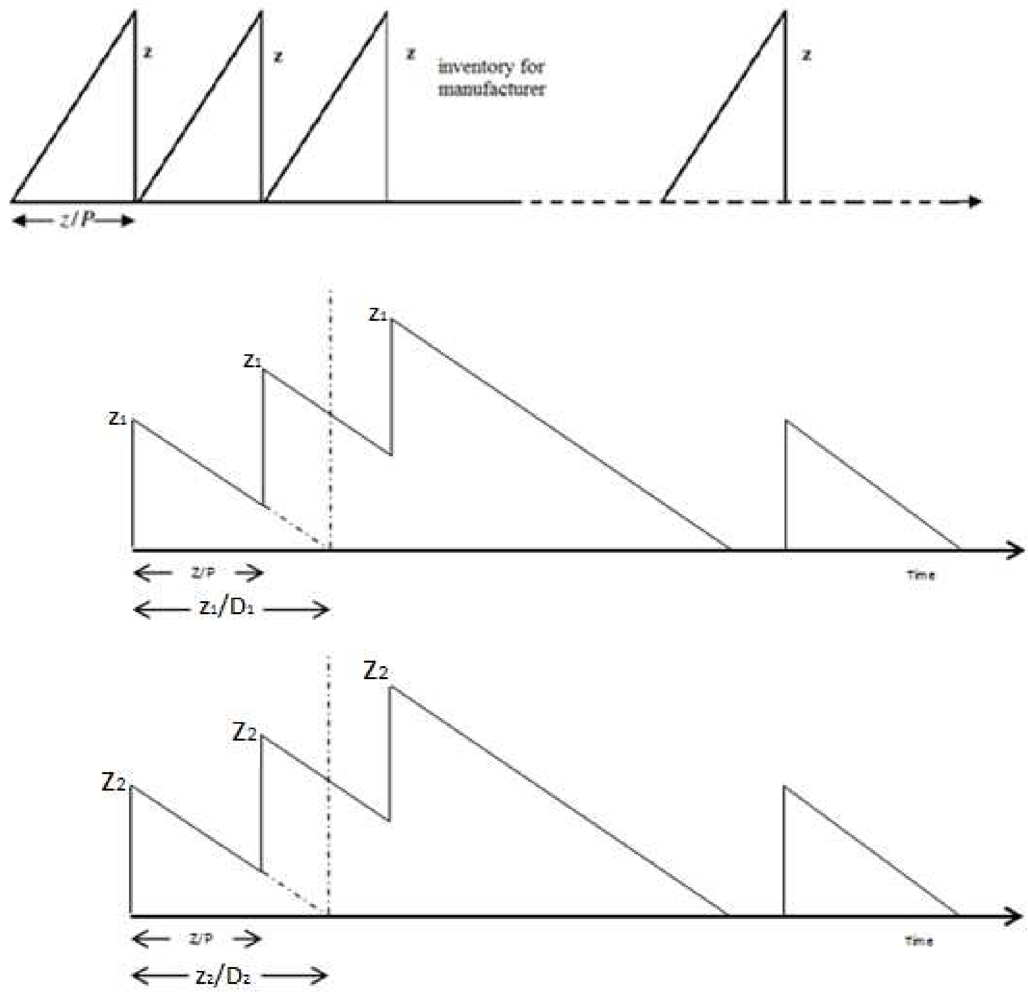

Figure 3.1: Inventory pattern for manufacturer and ith buyer

Manufaturer cost:

The manufacturer inventory cost is written in expression 3.3

$$\frac{Dhz}{2P} + \frac{DS}{nz} \tag{3.3}$$

Buyer cost:

The ordering cost added to transportation cost per year for the buyer i is $\frac{D(s_i + nT_i)}{nz}$. The total cost for each buyer will be:

$$\frac{zD_ih_i}{2D^2}+\frac{(n-1)}{2}\left(\frac{1}{D}-\frac{1}{P}\right)D_ih_iz+\frac{D(s_i+nT_i)}{nz} \qquad (3.4)$$

Total cost:

Adding 3.3 and 3.4 with simplification would lead to the following total cost function of the system note that we sum the cost for all m buyers:

$$TC=\left[\frac{Dh}{P}+\left\{\frac{1}{D}+(n-1)(\frac{1}{D}-\frac{1}{P})\right\}\sum_{i=1}^{m}D_ih_i\right]\frac{z}{2}+\frac{1}{z}\left[D\left\{\frac{S+\sum_{i=1}^{m}s_i}{n}+\sum_{i=1}^{m}T_i\right\}\right] \qquad (3.5)$$

3.1.3 Proposed relaxation to Hoque 2008

Hariga in [21] notes on generalized single-vendor multi-buyer integrated inventory supply chain models with better synchronization, the authors mentioned two possible relaxations to Hoque 2008:

[1]. The vendor could reduce his/her inventory holding costs by staggering the m sub-shipments over time instead of delivering them all at once.

[2]. Buyers could also make some cost savings by optimizing their lot sizes instead of fixing them as proportional to their relative demands.

3.1.3.1Relaxations to Model I of Hoque

Applying the above two relaxations to Hoque in [16] lead to two alternative solutions to the model. Since the vendor could send shipments to each buyer alone and the shipments were based on each buyer demand not the total demand of all buyers the vender could deliver the shipments cyclically or consecutively as shown in figure3.2 and figure 3.3.

So we called the cyclic policy as case I and the consecutive policy as case II.

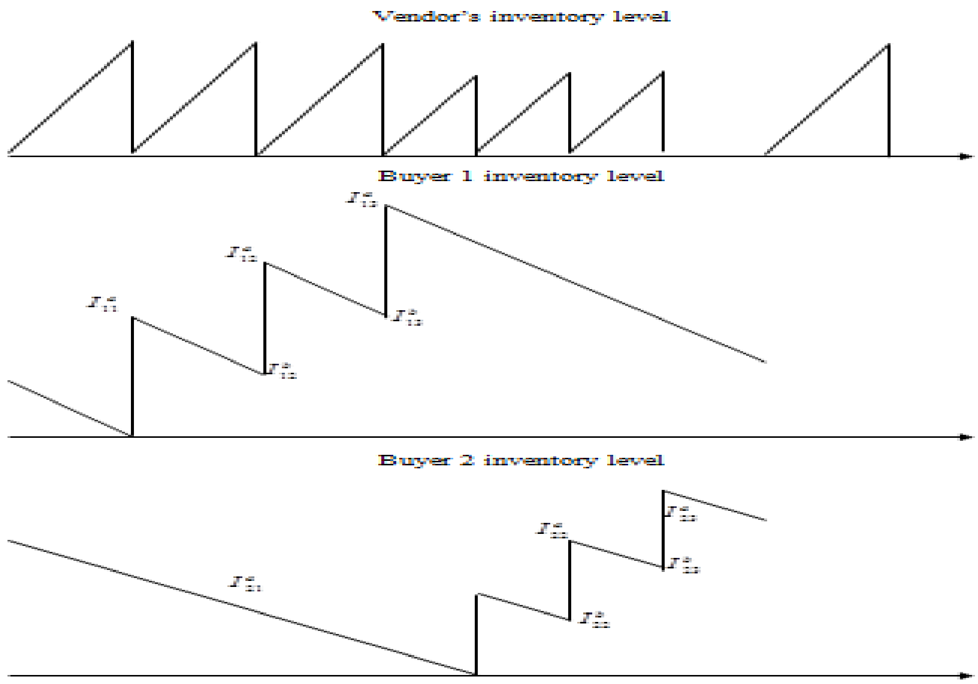

Figure 3. 2: Inventory profiles for vendor and buyers: consecutive policy

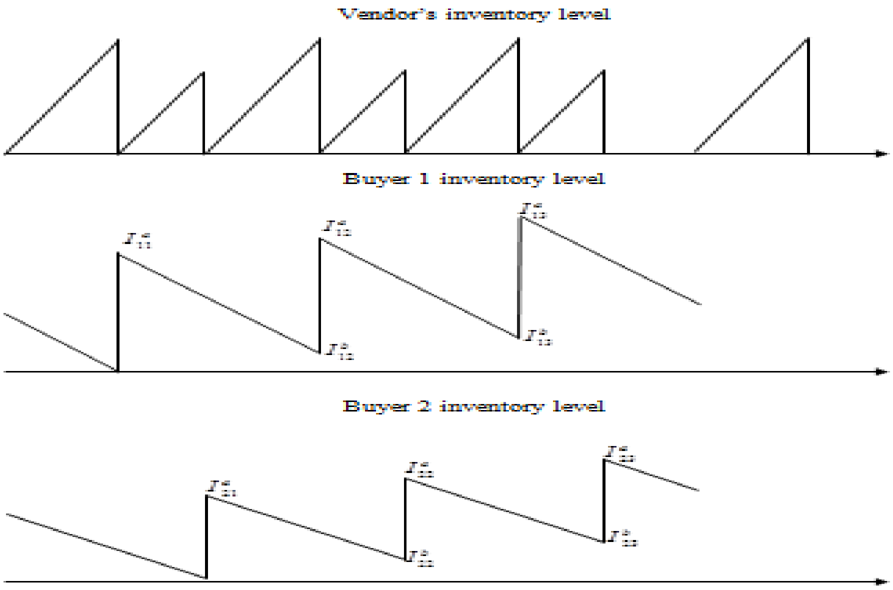

Figure 3.3: Inventory profiles for vendor and buyers: cyclic delivery policy

Case I model

The total cost function consisted of the setup, holding cost for the buyer and ordering, and holding cost of all the buyers. Hence:

$$TC^S = \frac{SD}{nz} + \frac{nz}{2D}\left[\frac{h}{nP}\sum_{i=1}^{m} D_i^2\right] + \sum_{i=1}^{m}\left[D\frac{nT_i + S_i}{nz} + \frac{nz}{2D}(1 - \frac{D}{P} + \frac{D}{nP})h_i D_i\right] \tag{3.6}$$

This can be rewritten as follows:

$$TC^S = \frac{S + \sum_{i=1}^{m}(nT_i + S_i)}{T} + \frac{T}{2}\left[\frac{h}{nP}\sum_{i=1}^{m} D_i^2 + (1 - \frac{D}{P} + \frac{D}{nP})\sum_{i=1}^{m} h_i D_i\right] \tag{3.7}$$

Using the first order optimality condition, the optimal cycle length is given by:

$$T_S^* = \sqrt{\frac{2\left[S + \sum_{i=1}^{m}(nT_i + S_i)\right]}{\left[\frac{h}{nP}\sum_{i=1}^{m} D_i^2 + (1 - \frac{D}{P} + \frac{D}{nP})\sum_{i=1}^{m} h_i D_i\right]}} \tag{3.8}$$

Substituting equation 3.8 into equation 3.7, we obtain the following expression:

$$TC^S = \sqrt{2\left[S+\sum_{i=1}^{m}(nT_i+S_i)\right]\left[\frac{h}{nP}\sum_{i=1}^{m}D_i^2 + (1-\frac{D}{P}+\frac{D}{nP})\sum_{i=1}^{m}h_iD_i\right]} \tag{3.9}$$

Case II model

Now let us consider the consecutive delivery policy. In this case, the total cost function is given by:

$$TC^C = \frac{S+\sum_{i=1}^{m}(nT_i+S_i)}{T} + \frac{T}{2}\left[\frac{h}{nP}\sum_{i=1}^{m}D_i^2 + \sum_{i=1}^{m}h_iD_i - \frac{\sum_{i=1}^{m}h_iD_i^2}{P}(1-\frac{1}{n})\right] \tag{3.10}$$

Using the first order optimality condition, the optimal cycle length is given by:

$$T_c^* = \sqrt{\frac{2\left[S+\sum_{i=1}^{m}(nT_i+S_i)\right]}{\frac{h}{nP}\sum_{i=1}^{m}D_i^2 + \sum_{i=1}^{m}h_iD_i - \frac{\sum_{i=1}^{m}h_iD_i^2}{P}(1-\frac{1}{n})}} \tag{3.11}$$

Substituting equation 3.11 into equation 3.10, the cost function can be rewritten as:

$$T_c^* = \sqrt{2\left[S+\sum_{i=1}^{m}(nT_i+S_i)\right]\left[\frac{h\sum_{i=1}^{m}D_i^2 + D_i^2}{nP} + \sum_{i=1}^{m}h_iD_i - \frac{\sum_{i=1}^{m}h_iD_i}{P}\right]} \tag{3.12}$$

3.1.4 Theoretical Comparison between Hoque Model and relaxed Models

In order to compare theoretically each part of total cost formula for each case when separated the cost pars as follows:

$$TC(HoqueModelI) = \frac{S + \sum_{i=1}^{m}(nT_i + S_i)}{T} + \frac{T}{2}\left[\frac{D^2 h}{nP} + \left\{1 - \frac{D}{P} + \frac{D}{nP}\right\}\sum_{i=1}^{m} D_i h_i\right]$$

$$TC^s(caseI) = \frac{S + \sum_{i=1}^{m}(nT_i + S_i)}{T} + \frac{T}{2}\left[\frac{h}{nP}\sum_{i=1}^{m} D_i^2 + (1 - \frac{D}{P} + \frac{D}{nP})\sum_{i=1}^{m} h_i D_i\right]$$

$$TC^c(CaseII) = \frac{S + \sum_{i=1}^{m}(nT_i + S_i)}{T} + \frac{T}{2}\left[\frac{h}{nP}\sum_{i=1}^{m} D_i^2 + \sum_{i=1}^{m} h_i D_i - \frac{\sum_{i=1}^{m} h_i D_i^2}{P}(1 - \frac{1}{n})\right]$$

A detail of the comparison is shown in Table 3.1.

Table 3.1: Comparision between extended models

Cost part	Hoque in [16] model I	Case I	Case II
Vendor setup cost	$\frac{S}{T}$	$\frac{S}{T}$	$\frac{S}{T}$
Buyer ordering cost	$\frac{\sum_{i=1}^{m}(S_i)}{T}$	$\frac{\sum_{i=1}^{m}(S_i)}{T}$	$\frac{\sum_{i=1}^{m}(S_i)}{T}$
Buyer transportation cost	$\frac{nT_i}{T}$	$\frac{nT_i}{T}$	$\frac{nT_i}{T}$
Vendor holding cost	$\frac{T}{2}\left[\frac{h}{nP}D^2\right]$	$\frac{T}{2}\left[\frac{h}{nP}\sum_{i=1}^{m} D_i^2\right]$	$\frac{T}{2}\left[\frac{h}{nP}\sum_{i=1}^{m} D_i^2\right]$
Buyer holding cost	$\frac{T}{2}\left[(1 - \frac{D}{P} + \frac{D}{nP})\sum_{i=1}^{m} h_i D_i\right]$	$\frac{T}{2}\left[(1 - \frac{D}{P} + \frac{D}{nP})\sum_{i=1}^{m} h_i D_i\right]$	$\frac{T}{2}\left[\sum_{i=1}^{m} h_i D_i - \frac{\sum_{i=1}^{m} h_i D_i^2}{P}(1 - \frac{1}{n})\right]$

It is clear that Hoque in [16] model I will cost much than its relaxation in case I, but comparing it with case II we cannot guarantee which one is best and that depends on the parameters.

3.1.5 Numerical Comparison

In this sub section, we considered the same numerical examples of Hoque in [16] and applied two relaxation models to the same examples. Two examples of Hoque were examined below.

3.1.5.1 Numerical Example1

In this numerical example, we supplied items to five buyers by a manufacturer, following the production and demand setups. Table 3.1 shows all data for buyers. Comparative results are shown in table 3.2.

$$S = 300, h = 0.2, P = 1500, D = \sum_{i=1}^{5} D_i = 970$$

Table 3.2: Data for example 1

i	S_i	D_i	h_i	T_i
1	25	200	0.22	25
2	15	150	0.24	20
3	25	225	0.25	18
4	30	230	0.23	25
5	30	165	0.21	15

Note that the in results tables (3.3), columns with label (1) are results of Hoque in [16] model I, columns with label (2) are results of Bendaya relaxation case I and columns with label (3) are result of case II respectively.

Table 3.3: Resuts for example1

Buyer	z_i (1)	z_i (2)	z_i (3)	Tc_i (1)	Tc_i (2)	Tc_i (3)
1	168.9471	219.1049	411.4041	75.27724	71.56012	69.56145
2	126.7103	164.3287	308.5531	59.4381	56.47463	54.04127
3	190.0654	246.493	462.8297	77.64907	76.62237	78.75773
4	194.2891	251.9706	473.1148	84.50068	81.4016	81.1459
5	139.3813	180.7615	339.4084	56.78374	55.21259	57.51418

	m(1)	m(2)	m(3)
z*	819.3932	1062.659	1995.31
n*	3.881204	2.98266	1.087471
n	4	3	1
T*	3.378941	3.286574	2.057021
nz	3277.573	3187.976	1995.31
TCv	141.7726	105.3945	172.3433
TC	495.4215	446.6658	513.3638

2.1.5.2 Numerical Example2

Let us consider the data in Table3.4 with the following setups. While Table 2.5 shows the comparative results.

$$S = 200, h = 3, P = 3250, D = \sum_{i=1}^{5} D_i = 1300$$

Table 3.4: Data for example2

i	S_i	D_i	h_i	T_i
1	25	300	3.1	25
2	15	250	3.2	20
3	25	200	3.15	18
4	30	225	3.25	25
5	30	325	3.1	15

Table 3.5: Results for example 2

z_i (1)	z_i (2)	z_i (3)	Tc_i (1)	Tc_i (2)	Tc_i (3)
76.72625	83.34446	132.0122	336.9063	341.6762	318.2447
63.93854	69.45371	110.0101	271.2078	276.7883	255.5543
51.15084	55.56297	88.00811	248.1552	249.804	236.331
57.54469	62.50834	99.00912	306.0164	306.5025	285.8783
83.12011	90.28983	143.0132	323.438	331.9044	323.9337

	m(1)	m(2)	m(3)
z*	332.4804	361.1593	572.0527
n*	2.010169	1.586381	0.74379
n	2	2	1
T*	0.511508	0.55563	0.440041
nz	664.9609	722.3186	572.0527
TCv	590.4887	404.6695	525.3332
TC	2076.212	1911.345	1945.275

From example 1 & 2 cases 1 of Bendaya relaxation, which was based on cyclic approach, it is clear that it leads to less cost. But in case 2, which was based on consecutive approach we could not guarantee better results.

3.2 Hoque [18] Model Extension

In this section, we considered Hoque's model in [18] where he sends mixed shipments between unequal shipments and equal shipments. The unequal shipment portion has an increasing factor equal to the ratio of production rate over the total buyers' demand rate, which was called K. In other words, Hoque combined two of his models in [16], which were model one and three as introduced in the literature.

Hoque in [18] sent n number of shipments to all buyers at the same time as previous models. The total number of shipments was divided to L unequal number of shipments and $(n - L)$ equal shipments. The unequal shipments are based on an increasing factor multiplied with z size of shipment and the equal shipments are of y size. Three cases of solutions were stated by the author. Case I when $L = 1$ the model becomes as model one of Hoque in [16], where he has equal shipments. In Case II, where he has $n = L$, the model becomes as model three of Hoque as in [16], where he has the unequal increasing shipments. However, Case III where he has $L > 1$ and $L < n$ *(that is $1<L<n$)*. Hoque in [16] clearly confirmed that model III was better than model I at all. Furthermore, solving various types of examples, case II was mostly giving better solution.

3.2.1 Hoque [18] Extension

While investigating Hoque's models, one could see that there were other restrictions in the models, rather than the two issues raised in Bendaya relaxation as discussed in the previous section, which were sending at the same time and not keeping each buyer to optimize his lot size depending on his demand. This two assumptions made the model carrying bigger inventory in each cycle and cost larger inventories for vendor and consequently for buyers. The other restriction made by Hoque that the shipment sizes were not free but it was restricted to a certain manner, which was increasing and equal.

In our extension we used the previous cyclic shipment model but with free size shipments for the same buyer each time of shipments. That is, if we have two buyers and two shipments to each buyer each shipment size to the same buyer is not dependent on other. We considered this strategy as free cyclic shipments.

3.2.2 Derivation of Extended Model

We used the same notations that we had in first chapter. The inventory pattern of this policy could be represented in Figure 3.4 below.

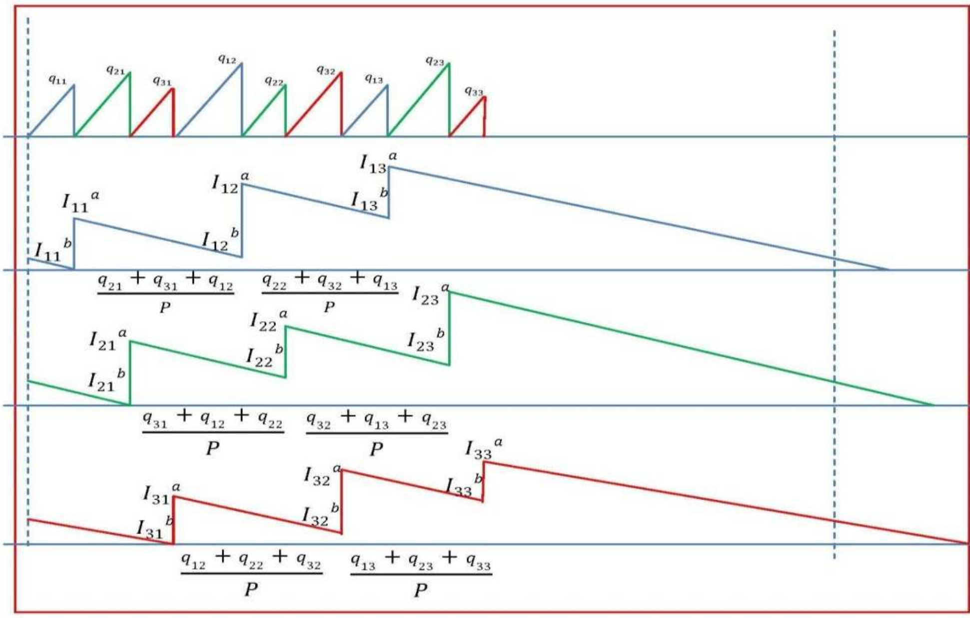

Figure 3.4: Extended model of free cyclic approach

n, is the total number of shipments to each buyer.

q_{ij}, is the unequal batch size produced by the vendor and shipped to buyer i immediately after production in the shipment number j. $1 \leq j \leq n$

Let $I_{ij}^{\ b}$ be the inventory level of the i^{th} buyer before receiving its j^{th} shipment, $1 \leq j \leq n$ and $I_{ij}^{\ b} = 0, i = 1, \ldots\ldots, m$

The total batch size shipped to buyer i is:

$$Q_i = \sum_{j=1}^{n} q_{ij}, i = 1, 2, 3, \ldots., m \tag{3.13}$$

But to avoid shortages:

$$Q_i = D_i T, i = 1, 2, 3, \ldots m \tag{3.14}$$

The total inventory for vendor will be:

$$\frac{1}{2P} \sum_{j=1}^{n} \sum_{i=1}^{m} q_{ij}^{\ 2} \tag{3.15}$$

As for the total inventory for buyer i :

We divided the area of buyers' inventory to two parts. The first part is marked with dark colors (1) and the second part gray (2) as shown in figure 3.5.

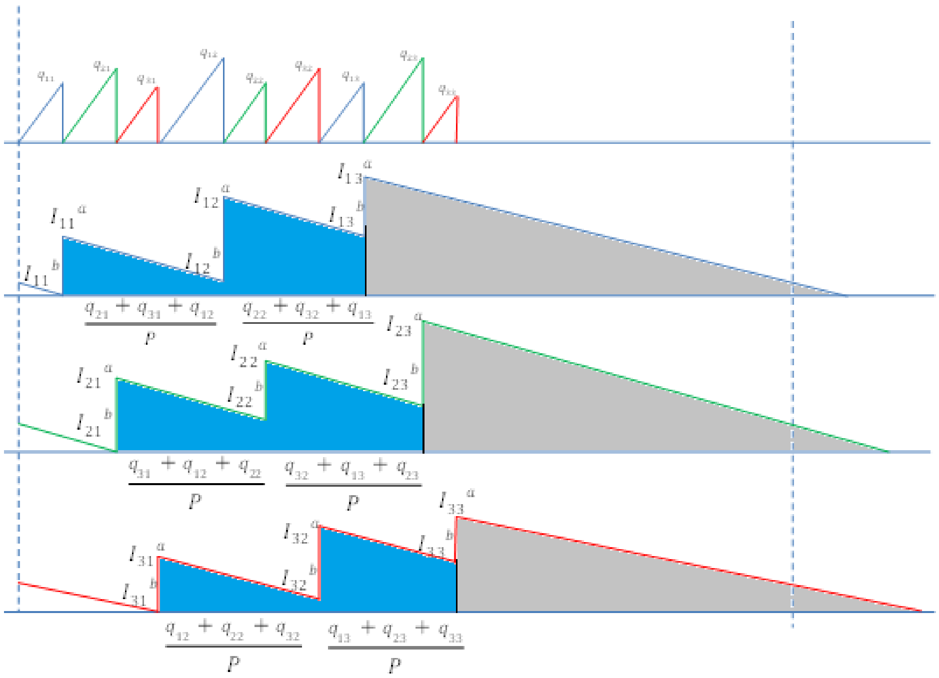

Figure 3.5: Free cyclic model divided areas

The area of the first part (marked as blue) is:

For buyer 1:

$$\left(I_{11}{}^{a}+I_{12}{}^{b}\right)\frac{q_{21}+q_{31}+\ldots+q_{m1}+q_{12}}{2P}+\left(I_{12}{}^{a}+I_{13}{}^{b}\right)\frac{q_{22}+q_{32}+\ldots+q_{m2}+q_{13}}{2P}+\left(I_{13}{}^{a}+I_{14}{}^{b}\right)\frac{q_{23}+q_{33}+\ldots+q_{m3}+q_{14}}{2P}+\ldots+\left(I_{1n-1}{}^{a}+I_{1n}{}^{b}\right)\frac{q_{2n-1}+q_{3n-1}+\ldots+q_{mn-1}+q_{1n}}{2P}$$

For buyer 2:

$$\left(I_{21}{}^{a}+I_{22}{}^{b}\right)\frac{q_{31}+q_{41}+\ldots+q_{m1}+q_{12}+q_{22}}{2P}+\left(I_{22}{}^{a}+I_{23}{}^{b}\right)\frac{q_{32}+q_{42}+\ldots+q_{m2}+q_{13}+q_{23}}{2P}+\left(I_{23}{}^{a}+I_{24}{}^{b}\right)\frac{q_{33}+q_{43}+\ldots+q_{m3}+q_{14}+q_{24}}{2P}$$

$$+\ldots+\left(I_{2n-1}{}^{a}+I_{2n}{}^{b}\right)$$

$$\frac{q_{3n-1}+q_{4n-1}+\ldots+q_{mn-1}+q_{1n}+q_{2n}}{2P}$$

For buyer 3:

$$\left(I_{31}{}^{a}+I_{32}{}^{b}\right)\frac{q_{41}+q_{51}+\ldots+q_{m1}+q_{12}+q_{22}+q_{32}}{2P}+\left(I_{32}{}^{a}+I_{33}{}^{b}\right)\frac{q_{42}+q_{52}+\ldots+q_{m2}+q_{13}+q_{23}+q_{33}}{2P}+\left(I_{33}{}^{a}+I_{34}{}^{b}\right)$$

$$\frac{q_{43}+q_{53}+\ldots+q_{m3}+q_{14}+q_{24}+q_{34}}{2P}+\ldots+\left(I_{3n-1}{}^{a}+I_{3n}{}^{b}\right)$$

$$\frac{q_{4n-1}+q_{5n-1}+\ldots+q_{mn-1}+q_{1n}+q_{2n}+q_{3n}}{2P}$$

Therefore, the general formula for fist part area is:

$$\sum_{j=1}^{n-1}\left(I_{ij}{}^{a}+I_{ij+1}{}^{b}\right)\frac{\sum_{x=i+1}^{m}q_{ij}+\sum_{x=1}^{i}q_{ij+1}}{2P}$$

Where:

$$I_{ij}{}^{a}=I_{ij}{}^{b}+q_{ij},\ j=1,2,3,\ldots,n,\ \ i=1,2,\ldots,m\ ,\ I_{11}{}^{b}=0\ ,$$

$$I_{12}{}^{b}=\left(q_{11}-\frac{q_{21}+q_{31}+\ldots+q_{m1}+q_{12}}{P}D_{1}\right),$$

$$I_{13}{}^{b}=\left(\left(q_{11}-\frac{q_{21}+q_{31}+\ldots+q_{m1}+q_{12}}{P}D_{1}\right)+q_{12}-\frac{q_{22}+q_{32}+\ldots+q_{m2}+q_{13}}{P}D_{1}\right)$$

$$=\left(q_{11}+q_{12}-\frac{q_{21}+q_{31}+\ldots+q_{m1}+q_{22}+q_{32}+\ldots+q_{m2}+q_{12}+q_{13}}{P}D_1\right),$$

$$I_{14}{}^{b}=\begin{pmatrix}\left(q_{11}+q_{12}-\dfrac{q_{21}+q_{31}+\ldots q_{m1}+q_{22}+q_{32}+\ldots q_{m2}+q_{12}+q_{13}}{P}D_1\right)+\\ q_{13}-\dfrac{q_{23}+q_{33}+\ldots q_{m3}+q_{14}}{P}D_1\end{pmatrix}$$

$$=\left(q_{11}+q_{12}+q_{13}-\frac{\begin{matrix}q_{21}+q_{31}+\ldots+q_{m1}+q_{22}+q_{32}+\ldots+q_{m2}+q_{23}+q_{33}+\ldots+\\ q_{m3}+q_{12}+q_{13}+q_{14}\end{matrix}}{P}D_1\right),$$

$$I_{1n}{}^{b}=\begin{pmatrix}q_{11}+q_{12}+q_{13}+\ldots+q_{1n-1}-\\ q_{21}+q_{31}+\ldots+q_{m1}+q_{22}+q_{32}+\ldots\\ +q_{m2}+q_{23}+q_{33}+\ldots+q_{m3}+\ldots+\\ \dfrac{q_{2n-1}+q_{3n-1}+\ldots+q_{mn-1}+q_{12}+q_{13}+q_{14}+\ldots+q_{1n}}{P}D_1\end{pmatrix},\ I_{21}{}^{b}=0,$$

$$I_{22}{}^{b}=\left(q_{21}-\frac{q_{31}+q_{41}+\ldots+q_{m1}+q_{12}+q_{22}}{P}D_1\right),$$

$$I_{23}{}^{b}=\left(\left(q_{21}-\frac{q_{31}+q_{41}+\ldots+q_{m1}+q_{12}+q_{22}}{P}D_1\right)+q_{22}-\frac{q_{32}+q_{42}+\ldots+q_{m2}+q_{13}+q_{23}}{P}D_1\right)$$

$$=\left(q_{21}+q_{22}-\frac{q_{31}+q_{41}+\ldots+q_{m1}+q_{32}+q_{42}+\ldots+q_{m2}+q_{12}+q_{22}+q_{13}+q_{23}}{P}D_1\right),$$

$$I_{24}{}^{b}=\begin{pmatrix}\left(q_{21}+q_{22}-\dfrac{q_{31}+q_{41}+\ldots+q_{m1}+q_{32}+q_{42}+\ldots+q_{m2}+q_{12}+q_{22}+q_{13}+q_{23}}{P}D_1\right)+\\ q_{23}-\dfrac{q_{33}+q_{43}+\ldots+q_{m3}+q_{14}+q_{24}}{P}D_1\end{pmatrix}$$

$$= \left(q_{21} + q_{22} + q_{23} - \frac{\begin{array}{c} q_{31} + q_{41} + \ldots + q_{m1} + q_{32} + q_{42} + \ldots + q_{m2} + q_{33} + q_{43} + \\ \ldots + q_{m3} + q_{12} + q_{22} + q_{13} + q_{23} + + q_{14} + q_{24} \end{array}}{P} D_1 \right),$$

$$I_{2n}^{b} = \left(\begin{array}{l} q_{21} + q_{22} + \ldots + q_{2n-1} - \\ \underline{\begin{array}{l} q_{21} + q_{31} + \ldots + q_{m1} + q_{22} + q_{32} + \ldots + q_{m2} + q_{23} + q_{33} + \\ \ldots + q_{m3} + \ldots + q_{2n-1} + q_{3n-1} + \ldots + q_{mn-1} + q_{12} + q_{13} + q_{14} \\ + \ldots + q_{1n} + q_{22} + q_{23} + q_{24} + .. + q_{2n} \end{array}} D_1 \\ \qquad\qquad\qquad P \end{array} \right), \text{ and}$$

$$I_{ij}^{\ b} = \left(\sum_{x=1}^{j-1} q_{ix} - \frac{\sum_{t=i+1}^{m} \sum_{x=1}^{j-1} q_{tx} + \sum_{x=2}^{j} \sum_{t=1}^{i} q_{tx}}{P} D_i\right), \quad j = 2,3,4,\ldots,n, \; i = 1,2,\ldots..m.$$

The area of the second part (marked as gray):

$$\frac{\left(I_{1n}^{\ a}\right)^2}{2D_1} + \frac{\left(I_{2n}^{\ a}\right)^2}{2D_2} + \ldots.. \frac{\left(I_{mn}^{\ a}\right)^2}{2D_m}$$

Therefore, the general formula will be:

$$\sum_{i=1}^{m} \frac{\left(I_{in}^{\ a}\right)^2}{2D_i}, \text{ for each } i = 1,2,\ldots..m$$

Thus, the combined formula for all buyers' inventory will be:

$$\frac{1}{2} \sum_{i=1}^{m} \left[\sum_{j=1}^{n-1} \left(I_{ij}^{\ a} + I_{ij+1}^{\ b}\right) \frac{\sum_{x=i+1}^{m} q_{ij} + \sum_{x=1}^{i} q_{ij+1}}{2P} + \sum_{i=1}^{m} \frac{\left(I_{in}^{\ a}\right)^2}{2D_i} \right] \tag{3.16}$$

$$I_{ij}^{\ a} = I_{ij}^{\ b} + q_{ij} \;, j = 1,2,3,\ldots n, \; i = 1,2,\ldots..m \tag{3.17}$$

$$I_{i1}^{\ b} = 0 \;, i = 1,2\;,3,\ldots.m$$

$$I_{ij}^{\ b} = (\sum_{x=1}^{j-1} q_{ix} - \frac{\sum_{t=i+1}^{m}\sum_{x=1}^{j-1} q_{tx} + \sum_{x=2}^{j}\sum_{t=1}^{i} q_{tx}}{P} D_i) \tag{3.18}$$
$$j = 2,3,4,\ldots,n, \quad i = 1,2,\ldots.m$$

The total cost per year for the system will be:

$$\sum_{i=1}^{m} \frac{Dh_i}{2Q}\left[\sum_{j=1}^{n-1}\left(I_{ij}^{\ a} + I_{ij+1}^{\ b}\right)\frac{\sum_{x=i+1}^{m} q_{ij} + \sum_{x=1}^{i} q_{ij+1}}{P} + \frac{\left(I_{in}^{\ a}\right)^2}{D_i}\right]$$
$$+\frac{Dh}{2QP}\sum_{j=1}^{n}\sum_{i=1}^{m} q_{ij}^{\ 2} + \frac{D}{Q}\sum_{i=1}^{m}\left(S + s_i + nT_i\right) \tag{3.19}$$

Subject to:

$$I_{ij}^{\ a} = I_{ij}^{\ b} + q_{ij} \ , j = 1,2,3,\ldots n, \ i = 1,2,\ldots.m \tag{3.20}$$

$$I_{i1}^{\ b} = 0 \ , i = 1,2\ ,3,\ldots.m$$

$$I_{ij}^{\ b} = (\sum_{x=1}^{j-1} q_{ix} - \frac{\sum_{t=i+1}^{m}\sum_{x=1}^{j-1} q_{tx} + \sum_{x=2}^{j}\sum_{t=1}^{i} q_{tx}}{P} D_i) \tag{3.21}$$
$$j = 2,3,4,\ldots,n, \quad i = 1,2,\ldots.m$$

$$Q_i = \sum_{j=1}^{n} q_{ij} \quad , i = 1,2,3,\ldots m$$

(3.22)

$$Q_i = D_i T \quad , i = 1,2,\ldots.m \quad (2.19)$$

$$Q = \sum_{i=1}^{m} Q_i \tag{3.23}$$

$$I_{ij}^{\ b} \geq 0$$
$$for\ j = 1,2,3,\ldots\ldots n \ , i = 1,2,\ldots.m \tag{3.24}$$

3.2.3 Importing Settings to Get Better Results

In order to get better results for the extended model one has to reorder the buyers in the following criteria:

- First reorder the buyers in increasing buyer holding cost.
- If the buyer holding costs are equal reorder the buyer who has equal holding cost in decreasing demands.

3.2.4 Comparative Studies

To compare the model of Hoque in [18] and our extended model we used the same as the first example he used in his paper. We used sub examples of his 5 buyer examples and beside the 5 buyer example. Later we generated two other larger examples of 10 and 15 buyers' problems and showed the improvements. Therefore, in conclusion we have 9 examples for comparison.

Example1:

Table 3.6: Data for example 1(3 buyers)

S	400	i	Si	Di	Hi	Ti
h	4	1	25	200	7	25
P	3200	2	15	180	6	20
D	605	3	25	225	7.5	18

Table 3.7: Results for example1

Hoque [18]		Free Cyclic Model	
n	2	n	2
L	2		
z	56.81541		
y	0		
T*	0.590649	T*	0.59279
Q	357.3429	Q	358.638
TCv	776.2045	TCv	710.4232
TC	2001.187	TC	1999.126

Example 2

Table 3.8: Data for example 2

S	400	i	Si	Di	Hi	Ti
H	4	1	25	200	7	25
P	3200	2	25	225	7.5	18
D	655	3	30	230	6.5	25

Table 3.9: Results for example 2

Hoque [18]		Free Cyclic Model	
n	2	N	2
L	2		
z	64.43802		
y	0		
T*	0.579007	T*	0.590621
Q	379.2497	Q	386.8568
TCv	802.2986	TCv	718.3029
TC	2127.78	TC	2126.552

Example 3

Table 3.10: Data for example 3

S	400	i	Si	Di	Hi	Ti
h	4	1	30	165	6	15
P	3200	2	25	200	7	25
D	595	3	30	230	6.5	25

Table 3.11: Results for example 3

Hoque [18]		Free Cyclic Model	
N	2	n	2
L	2		
Z	57.98768		
Y	0		
T*	0.621604	T*	0.62747
Q	369.8542	Q	373.3446
TCv	744.6699	TCv	674.4954
TC	1978.752	TC	1976.257

Example 4

Table 3.12: Data for example 4

S	400	I	Si	Di	Hi	Ti
h	4	1	30	165	6	15
P	3200	2	15	180	6	20
D	770	3	25	200	7	25
k	4.155844	4	25	225	7.5	18

Table 3.13: Results for example 4

Hoque [18]		Free Cyclic Model	
N	2	n	2
L	2		
Z	84.63287		
Y	0		
T*	0.566693	T*	0.575362
Q	436.3539	Q	443.0289
TCv	850.1846	TCv	735.3959
TC	2297.538	TC	2277.757

Example 5

Table 3.14: Data for example 5

S	400	i	Si	Di	Hi	Ti
h	4	1	30	165	6	15
P	3200	2	15	180	6	20
D	800	3	25	225	7.5	18
k	4	4	30	230	6.5	25

Table 3.15: Results for example 5

Hoque [18]		Free Cyclic Model	
N	2	n	2
L	2		
Z	90.33687		
Y	0		
T*	0.564605	T*	0.55849
Q	451.6843	Q	446.7918
TCv	862.032	TCv	758.1167
TC	2323.747	TC	2300.504

Example 6

Table 3.16: Data for example 6

S	400	i	Si	Di	Hi	Ti
h	4	1	15	180	6	20
P	3200	2	25	200	7	25
D	835	3	25	225	7.5	18
k	3.832335	4	30	230	6.5	25

Table 3.17: Results for example 6

Hoque [18]		Free Cyclic Model	
N	2	n	2
L	2		
Z	95.54438		
Y	0		
T*	0.552937	T*	0.537497
Q	461.7025	Q	448.8104
TCv	885.273	TCv	787.4404
C	2427.04	TC	2403.521

Example 7

Table 3.18: Data for example 7

S	400	i	Si	Di	Hi	Ti
h	4	1	25	200	7	25
P	3200	2	15	180	6	20
D	1000	3	25	225	7.5	18
k	3.2	4	30	230	6.5	25
		5	30	165	6	15

Table 3.19: Results for example 7

<table>
<tr><th colspan="2">Hoque [18]</th><th colspan="2">Free Cyclic Model</th></tr>
<tr><td>N</td><td>3</td><td rowspan="4">n</td><td rowspan="4">2</td></tr>
<tr><td>L</td><td>3</td></tr>
<tr><td>Z</td><td>42.63872</td></tr>
<tr><td>Y</td><td>0</td></tr>
<tr><td>T*</td><td>0.615703</td><td>T*</td><td>0.56494</td></tr>
<tr><td>Q</td><td>615.7031</td><td>Q</td><td>564.9405</td></tr>
<tr><td>TCv</td><td>863.9233</td><td>TCv</td><td>758.0454</td></tr>
<tr><td>TC</td><td>2709.098</td><td>TC</td><td>2654.947</td></tr>
</table>

Example 8

Table 3.20: Data for example 8

S	400	i	Si	Di	Hi	Ti
h	4	1	25	200	7	25
P	3200	2	15	180	6	20
D	1950	3	25	225	7.5	18
k	1.641026	4	30	230	6.5	25
		5	30	165	6	15
		6	25	150	7.5	17
		7	10	185	5	16
		8	25	190	5.5	20
		9	15	200	6.5	15
		10	35	225	6	10

Table 3.21: Results for example 8

<table>
<tr><th colspan="2">Hoque [18]</th><th colspan="2">Free Cyclic Model</th></tr>
<tr><td>N</td><td>4</td><td rowspan="4">n</td><td rowspan="4">3</td></tr>
<tr><td>L</td><td>4</td></tr>
<tr><td>Z</td><td>140.6211</td></tr>
<tr><td>Y</td><td>0</td></tr>
<tr><td>T*</td><td>0.703337</td><td>T*</td><td>0.642106</td></tr>
<tr><td>Q</td><td>1371.508</td><td>Q</td><td>1252.107</td></tr>
<tr><td>TCv</td><td>1104.212</td><td>TCv</td><td>691.3839</td></tr>
<tr><td>TC</td><td>3864.432</td><td>TC</td><td>3636.691</td></tr>
</table>

Example 9

Table 3.22: Data for example 9

S	400	i	Si	Di	Hi	Ti
h	4	1	25	200	7	25
P	3900	2	15	180	6	20
D	2940	3	25	225	7.5	18
k	1.326531	4	30	230	6.5	25
		5	30	165	6	15
		6	25	150	7.5	17
		7	10	185	5	16
		8	25	190	5.5	20
		9	15	200	6.5	15
		10	35	225	6	10
		11	30	165	4.5	12
		12	25	180	6.3	17
		13	40	190	7.5	23
		14	25	240	7	19
		15	20	215	6.5	21

Table 3.23: Results for example 9

Hoque [18]		Free Cyclic Model	
N	6	n	4
L	6		
Z	199.8991		
Y	0		
T*	0.926373	T*	0.790489
Q	2723.537	Q	2324.038
TCv	1267.195	TCv	579.6625
TC	5209.563	TC	4715.937

Summary of Comparison

We had summarized the examples in the following table 3.24.

Table 3.24: Summary of comparisons

			Hoque [18]						Free cyclic model				comparison results	
problem	number of buyers	K	n,L	Z	y	TCbi	TCv	TC	N	TCbi	TCv	TC	cost reduction	% Improvement
1	3	5.289256	2,3	56.815	0	1224.98	776.204	2001.19	2	1288.7	710.42	1999.13	2.0613	0.10%
2	3	4.885496	2,4	64.438	0	1325.48	802.299	2127.78	2	1408.25	718.3	2126.55	1.22764	0.06%
3	3	5.378151	2,5	57.988	0	1234.08	744.67	1978.75	2	1301.76	674.5	1976.26	2.49481	0.13%
4	4	4.155844	2,6	84.633	0	1447.35	850.185	2297.54	2	1542.36	735.4	2277.76	19.7815	0.86%
5	4	4	2,7	90.337	0	1461.71	862.032	2323.75	2	1542.39	758.12	2300.5	23.2423	1.00%
6	4	3.832335	2,8	95.544	0	1541.77	885.273	2427.04	2	1616.08	787.44	2403.52	23.5192	0.97%
7	5	3.2	3,3	42.639	0	1845.17	863.923	2709.1	2	1896.9	758.05	2654.95	54.1514	2.00%
8	10	1.641026	4,4	140.62	0	2760.22	1104.21	3864.43	3	2945.31	691.38	3636.69	227.742	5.89%
9	15	1.326531	6,6	199.9	0	3942.37	1267.19	5209.56	4	4136.27	579.66	4715.94	493.626	9.48%

As we saw in the summary table as the number of buyers increased the percentage of improvement was increasing. In the example where there were three buyers, the improvements were under 0.2 % but in four buyers examples the improvements were around 1%. 2% was for 5 buyer's example but around 6% percent for 10 buyers. Finally, for 15 buyers the improvement was around 10%. The figure 3.6 will show the pattern of the change in improvements in results.

Figure 3.6: Improvement diagram as number of buyers increase

CHAPTER 4

Extension of Cyclic Shipping to Buyers Approach

4.1 Bendaya 2012 Model

Bendaya et al. in [6] introduces the idea of consignment (CS) and vendor managed inventory (VMI). Thus, the idea was based on managing inventory between vendor and buyers such that the total cost is minimized. They studied three types of models. The first was an arrangement such that the vendor and buyers act independently. The second was where the vendor entered into a vendor managed inventory consignment partnership with buyers. The third arrangement was where the inventory was centralized which meant that the cost was shared equally between vendor and buyers.

Bendaya used the cyclic policy mentioned earlier where the vendor sends a shipment for the first buyer and then a shipment to the second then the rest. Then the vendor returns to the first buyer to send him the second shipment and so on. Each shipment would last until the next shipment is received. In his model, equal number of shipments are sent to all buyers i.e. he would send n shipments to each buyer. Also, the shipments to each buyer were equal sized. Here we have rewritten the Bendaya Model in terms of notations stated in this paper for the purpose of clarity. Figure 4.1 shows the inventory pattern.

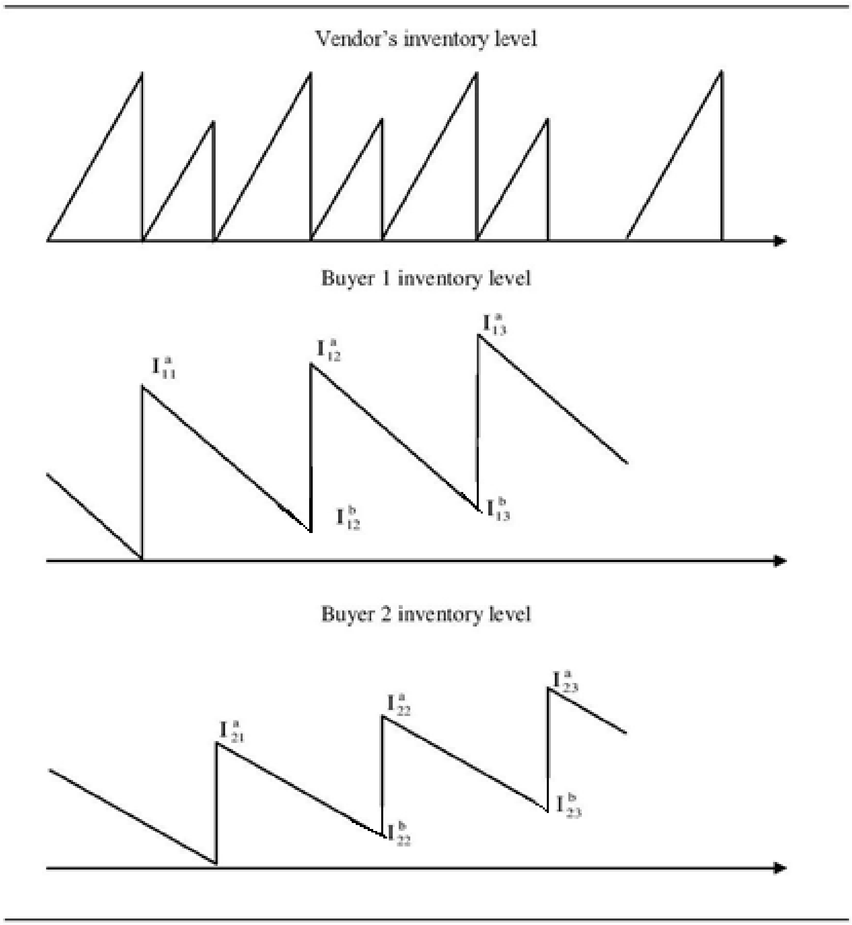

Figure 4.1: Inventory profiles for one vendor and two buyers

The total cost function consists of the setup, holding cost for the buyer and ordering, and holding cost of all the buyers. Hence:

$$\mathrm{TC} = \frac{\mathrm{S}}{T} + \frac{T}{2}\left[\frac{\mathrm{h}}{nP}\sum_{i=1}^{N} D_i^2\right] + \sum_{i=1}^{N}\left[\frac{nS_i}{T} + \frac{T}{2}\left(1 - \frac{D}{P} + \frac{D}{nP}\right)h_i D_i\right] \tag{4.1}$$

This can be rewritten as follows:

$$\mathrm{TC} = \frac{\mathrm{S} + n\sum_{i=1}^{N} S_i}{T} + \frac{T}{2}\left[\frac{\mathrm{h}}{nP}\sum_{i=1}^{N} D_i^2 + \left(1 - \frac{D}{P} + \frac{D}{nP}\right)\sum_{i=1}^{N} h_i D_i\right] \tag{4.2}$$

Using the first order optimality condition, the optimal cycle length is given by:

$$T^{*} = \sqrt{\frac{2\left[\mathrm{S} + n\sum_{i=1}^{N} S_i\right]}{\frac{\mathrm{h}\sum_{i=1}^{N} D_i^2}{nP} + \left(1 - \frac{D}{P} + \frac{D}{nP}\right)\sum_{i=1}^{N} h_i D_i}} \tag{4.3}$$

Substituting equation 4.2 in equation 4.1, we obtain the following expression:

$$\mathrm{TC} = \sqrt{2\left[\mathrm{S} + n\sum_{i=1}^{N} S_i\right]\left[\frac{\mathrm{h}}{nP}\sum_{i=1}^{N} D_i^2 + \left(1 - \frac{D}{P} + \frac{D}{nP}\right)\sum_{i=1}^{N} h_i D_i\right]} \tag{4.4}$$

Then, solving for *n* knowing that *n* should be integer, we obtain the following expression for the number of shipment to be sent to each buyer:

$$\mathrm{n}^{*} = 0.5(-1 + \sqrt{1 + 4\mathrm{S}\frac{\mathrm{h}\sum_{i=1}^{N} D_i^2 + D\sum_{i=1}^{N} h_i D_i}{(P-D)\sum_{i=1}^{N} S_i \sum_{i=1}^{N} h_i D_i}} \tag{4.5}$$

4.2 Extension on Bendaya [6] Model

To relax Bendaya model we let the vendor sends independent shipments to each buyer cyclically. Bendaya was sending equal shipments to each buyer but not equal to all buyers. This policy may not optimize the model because the vendor should hold more inventories compulsorily and not the amount the buyer need in each shipment. Consequently, the vendor would hold more inventory and the buyer also hold more.

Figure 4.2 shows the pattern of the inventory and how the shipments were managed to each buyer:

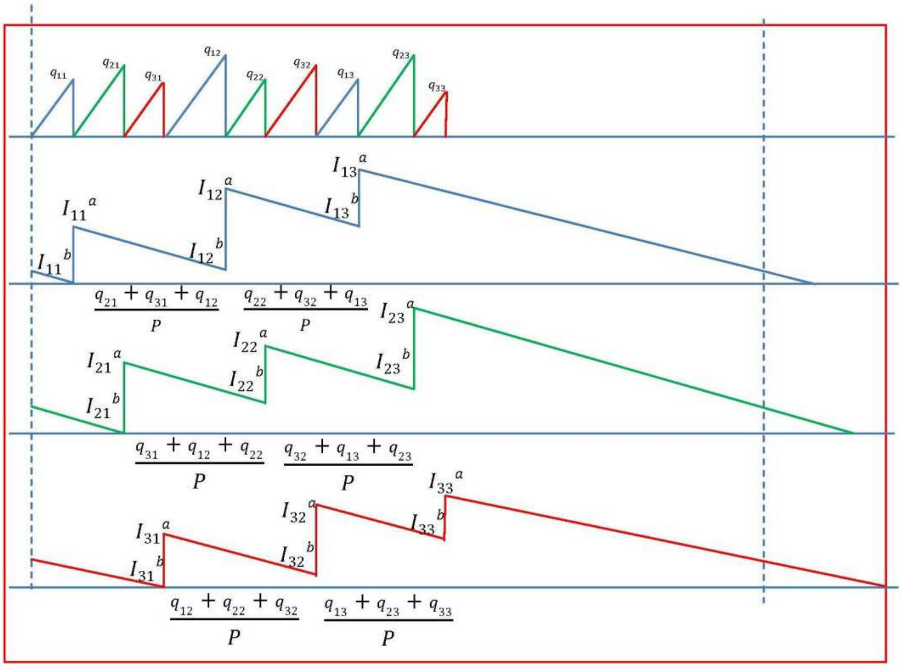

Figure 4.2: The inventory pattern for the extended model

Where,

n, is the total number of shipments to each buyer.

q_{ij}, is the unequal batch size produced by the vendor and shipped to buyer i immediately after production in the shipment number j. $1 \le j \le n$

Let $I_{ij}{}^{b}$ be the inventory level of the i^{th} buyer before receiving its j^{th} shipment, $1 \le j \le n$ and $I_{i1}{}^{b} = 0$,$i = 1, \ldots, m$

Let $I_{ij}{}^{a}$ be the inventory level of the i^{th} buyer after receiving its j^{th} shipment, $1 \le j \le n_i$.

The rest notations were the same as we introduced in chapter one.

The total cost of the system per year will be as follows:

$$\sum_{i=1}^{m}\frac{Dh_i}{2Q}\left[\sum_{j=1}^{n-1}\left(I_{ij}{}^{a}+I_{ij+1}{}^{b}\right)\frac{\sum_{x=i+1}^{m}q_{ij}+\sum_{x=1}^{i}q_{ij+1}}{P}+\frac{\left(I_{in}{}^{a}\right)^2}{D_i}\right]+$$
$$\frac{Dh}{2QP}\sum_{j=1}^{n}\sum_{i=1}^{m}q_{ij}{}^{2}+\frac{D}{Q}\sum_{i=1}^{m}\left(S+ns_i\right) \tag{4.6}$$

Subject to:

$$I_{ij}{}^{a}=I_{ij}{}^{b}+q_{ij}\ ,j=1,2,3,\ldots n,\ \ i=1,2,\ldots..m \tag{4.7}$$

$$I_{i1}{}^{b}=0\ ,i=1,2\ ,3,\ldots.m \tag{4.8}$$

$$I_{ij}{}^{b}=(\sum_{x=1}^{j-1}q_{ix}-\frac{\sum_{t=i+1}^{m}\sum_{x=1}^{j-1}q_{tx}+\sum_{x=2}^{j}\sum_{t=1}^{i}q_{tx}}{P}D_i)$$
$$j=2,3,4,\ldots,n,\ \ i=1,2,\ldots,m \tag{4.9}$$

$$Q_i=\sum_{j=1}^{n}q_{ij}\quad ,i=1,2,3,\ldots.m \tag{4.10}$$

$$Q_i=D_iT\quad ,i=1,2,\ldots..m \tag{4.11}$$

$$Q=\sum_{i=1}^{m}Q_i \tag{4.12}$$

$$I_{ij}{}^{b}\geq 0\ \ for\ j=1,2,3,\ldots\ldots n\ ,i=1,2,\ldots..m \tag{4.13}$$

Equations 4.7 – 4.13 are the constraints to the problem. With equations 4.7 – 4.9 as constraints of the definition of the inventory levels, equation 4.10 is the constraint that defines the total quantity shipped to each buyer among n shipments. The equation 4.11 constraint is avoiding shortages at end of each cycle and starting period. The equation 4.12 constraint defines Q in the objective function. Finally, equation 4.13 constraint is avoiding shortages between shipments for each buyer.

4.3 Numerical Comparison

We have solved 6 different examples in order to evaluate the improvements of our new model when compared to those in the literature. The first example was Zavanella [32] example of two buyers that was later solved in Bendaya in [6]. The rest examples were larger examples of 3, 4, 5, 10 and 15 buyers. In each example, we stated the data and complete solution. In the last part of the section, we compare all results in a table.

4.3.1 Examples and Solutions

Example1

Table 4.1: Data for example 1

S	400	I	Si	Di	hi
h	5	1	75	500	4
P	3200	2	25	1000	4
D	1500				
K	2.133333				

Table 4.2: Results for example 1

Bendaya [6] model		Extended Model	
z*	270.7072		
n*	2.445483		
N	3	n	3
T*	0.541414	T*	0.623236
Q	812.1215	Q	934.8537
TCv	915.0473	TCv	937.8059
TC	2585.819	TC	2313.661

Example 2

Table 4.3: Data for example 2

S	400	I	Si	Di	hi
h	5	1	100	200	3
P	3200	2	25	1000	4
D	1700	3	75	500	4
k	1.882353				

Table 4.4: Results for example 2

Bendaya [6] model		Extended Model	
z*	444.3518		
n*	1.889365		
N	2	n	3
T*	0.522767	T*	0.718599
Q	888.7036	Q	1221.618
TCv	1028.585	TCv	879.2421
TC	3060.637	TC	2780.217

Example 3

Table 4.5: Data for example 3

S	400	I	Si	Di	hi
h	5	1	100	200	3
P	3200	2	25	1000	4
D	2100	3	75	500	4
k	1.52381	4	50	400	5

Table 4.6: Results for example 3

Bendaya [6] model		Extended Model	
z*	535.867		
n*	2.068998		
N	2	n	2
T*	0.510349	T*	0.557256
Q	1071.734	Q	1170.238
TCv	1072.842	TCv	1056.122
TC	3526.994	TC	3234.459

Example 4

Table 4.7: Data for example 4

S	400	I	Si	Di	hi
h	5	1	100	200	3
P	3200	2	75	500	4
D	2800	3	25	1000	4
k	1.142857	4	80	700	4.5
		5	50	400	5

Table 4.8: Results for example 4

Bendaya [6] model		Extended Model	
z*	640.3291		
n*	3.314583		
N	3	n	6
T*	0.686067	T*	1.318575
Q	1920.987	Q	3692.009
TCv	929.6402	TCv	672.205
TC	4052.083	TC	3607.062

Example 5

Table 4.9: Data for example 5

S	400	I	Si	Di	hi
h	5	1	75	500	4
P	10000	2	25	1000	4
D	6470	3	100	200	3
k	1.545595	4	50	400	5
		5	80	700	4.5
		6	70	460	4.5
		7	90	820	6.5
		8	45	750	7
		9	95	910	4
		10	120	730	5.5

Table 4.10: Results for example 5

Bendaya [6] model		Extended Model	
z*	1672.275		
n*	1.043748		
N	1	n	2
T*	0.258466	T*	0.44952
Q	1672.275	Q	2908.394
TCv	1854.36	TCv	1183.133
TC	8898.659	TC	8425.992

Example 6

Table 4.11: Data for example 6

S	400	I	Si	Di	hi
h	5	1	100	200	3
P	12000	2	25	1000	4
D	10292	3	95	910	4
k	1.165954	4	75	500	4
		5	45	600	4.9
		6	68	1300	4.6
		7	80	700	4.5
		8	70	460	4.5
		9	50	400	5
		10	72	792	5.2
		11	120	730	5.5
		12	83	480	6.4
		13	90	820	6.5
		14	45	750	7
		15	35	650	7.5

Table 4.12: Results for example 6

Bendaya [6] model		Extended Model	
z*	2037.283		
n*	1.56791		
N	2	n	3
T*	0.395896	T*	0.618313
Q	4074.566	Q	6363.678
TCv	1343.482	TCv	1008.4
TC	12659.88	TC	11778.23

4.3.2 Examples' Results and Comparison

We can now summarize the results of the comparison in the following table 4.13.

Table 4.13: Comparison Results

		Bendaya [6] Solution		Relaxed Model		% of Improvement
ex#	number of buyers	number of shipments	Total Cost	number of shipments	Total Cost	
1	2	3	2585.818697	3	2313.660505	10.53%
2	3	2	3060.637189	3	2780.217396	9.16%
3	4	2	3526.994117	2	3234.458771	8.29%
4	5	3	4052.082798	6	3607.061755	10.98%
5	10	1	8898.658607	2	8425.992161	5.31%
6	15	2	12659.87791	3	11778.22539	6.96%

As seen in the table 4.13, 5.31% is recorded as the minimum improvement, and the maximum improvement was 10.98%, which were good results and concluded that the extended model was a better solution and fitted more for the supply chain relation between vendor and buyers.

CHAPTER 5

Efficient Heuristics to Solve General Models

5.1 Hariga [20] General Model

Hariga et al. in [20] proposed a general model for the single vendor multi-buyer problem. The model was an extension of Ben-Daya's model in [6] by removing several restrictions including the equal size and number of shipments restrictions. It provides the most general model and yields a better solution than all key models in literature. Solving the general problem was hard, as it requires finding optimal delivery schedule to the buyers and optimal production lot sizes. Hariga et al [20] provided a lower bound for the general problem and then proposed a heuristic procedure to generate a near optimal delivery schedules. Figure 5.1 shows the pattern of inventory for the vendor.

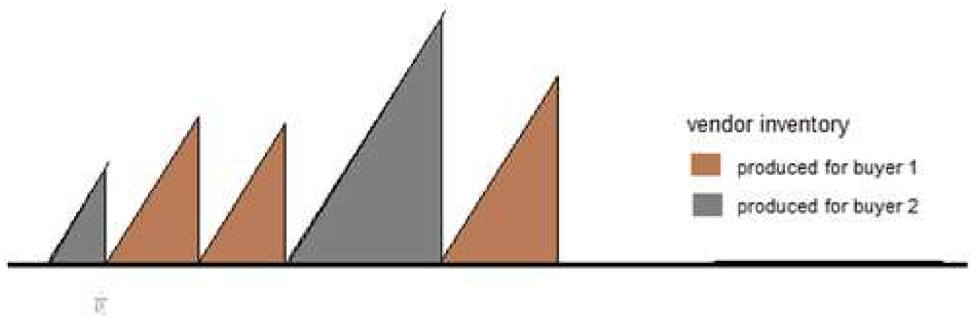

Figure 5.1: Inventory pattern for the vendor.

The presented heuristic procedure depends on certain conditions called the "zero switch rule" (ZSR). ZSR stated that each shipment for a buyer would be consumed fully just before the next shipment to the same buyer arrives. So there were zero inventories when the shipments were received and there would be no shortages. Figure 5.2 shows the inventory pattern for an example involving two buyers under the ZSR.

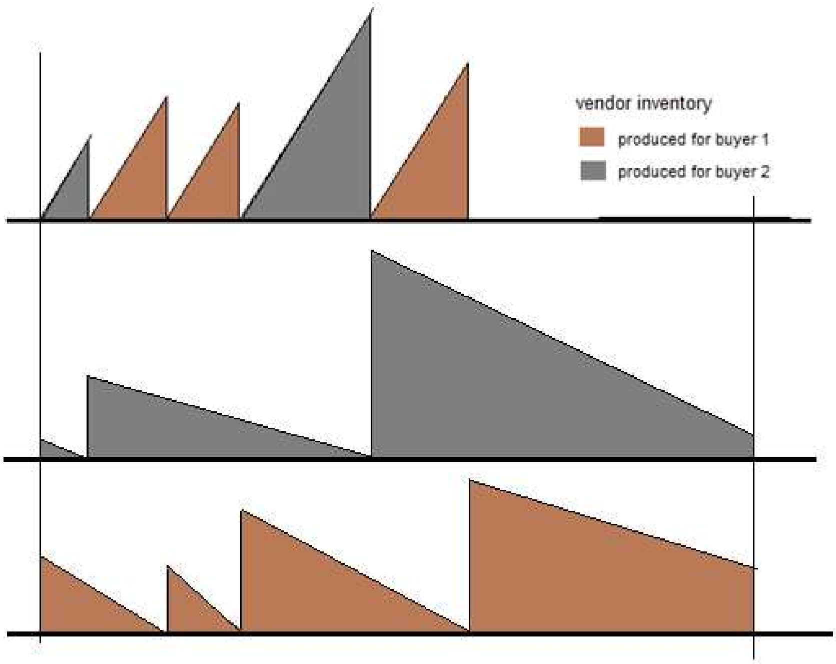

Figure 5.2: Inventory pattern for zero switch rule model

The authors proposed a heuristic to solve the relaxed ZSR model. The proposed method determine the number of shipments to each buyer based on assuming that the vendor is dealing with only one buyer and finding the corresponding optimal number of shipments. The proposed heuristic first determine the best sequence of the shipments to the buyer using bin packing method. Then a quadratic program is solved to obtain the timing of these shipments.

The zero switch rule problem formulation is stated below in terms of the notations stated in this thesis.

5.1.1 Zero Switch Rule General Model Formulation

Let

i: is the buyer index

j: is the shipment or delivery order number.

m_i : Number of shipments sent to buyer i in the cycle.

t_j : is the time produce the quantity shipped in the jth delivery.

: time interval between the jth and (j+1) the replenishment

$$u_j = \begin{cases} t_{j+1}, j=1,2,....m-1 \\ t_1+t_{j+1}, j=m \end{cases}$$

$$Pt_{ij}x_{ij} = q_{ij}$$

x_{ij} =1 if the jth shipment is sent to *ith* buyer and 0 otherwise.

y_{ijk} =1 when the next delivery to the *i*th buyer after *j*th replenishment is made at the *k*th shipment and is zero otherwise.

T_{ij}, is time to send the next shipment to buyer *i* after the jth shipment.

T_j, is time to send the next shipment to same buyer after the jth shipment.

The formulation of the problem is as follows:

min

$$ATC_{ZSR} = \frac{(S + \sum_{i=1}^{n} m_i S_i + 0.5 \sum_{j=1}^{m} T_j^{\,2} \left[\sum_{i=1}^{n} h_i D_i x_{ij} + \frac{h}{P} \sum_{i=1}^{n} D_i^{\,2} x_{ij} \right])}{T} \tag{5.1}$$

$s.t.$

$$\sum_{j=1}^{m} x_{ij} = m_{i,} i = 1,2,3,......n \tag{5.2}$$

$$\sum_{i=1}^{n} x_{ij} = 1, i = 1,2,3,......m \tag{5.3}$$

$$\sum_{k=1}^{m} y_{ijk} = 1, i = 1,2,3,......n \text{ and } j = 1,2,3,....m \tag{5.4}$$

$$y_{ijk} - x_{ik} \le 0, i = 1,2,3,...,n \text{ And } j = 1,2,....m \tag{5.5}$$

$$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl} \quad i = 1,2,....n \text{ And } j,k = 1,2,....,m \tag{5.6}$$

$$Y_{ijk} \ge x_{ik}, i = 1,2,....n \text{ And } j,k = 1,2,.....m \tag{5.7}$$

$$T_{ij} = u_j + \sum_{k=j=1}^{j-1} (1 - Y_{ijk}) u_k \quad i = 1,2,.....n \text{ And } j = 1,2,.....m \tag{5.8}$$

$$\sum_{j=1}^{m} u_j = T, j = 1,2....m \tag{5.9}$$

$$T_j = \sum_{i=1}^{n} T_{ij} x_{ij} forj = 1,2,....m \tag{5.10}$$

$$Pu_j = T_{j+1} \sum_{i=1}^{n} D_i x_{ij+1} forj = 1,2,....m-1 \tag{5.11}$$

$$Pu_m = T_1 \sum_{i=1}^{n} D_i x_{i1} + \left(P - \sum_{i=1}^{n} D_i \right) T \tag{5.12}$$

$$u_j \ge 0, T_j \ge 0, T_{ij} \ge 0, y_{ijk,} x_{ij} = 0 or 1; i = 1,....n \text{ And } j = 1,....m \tag{5.13}$$

Equation (5.1) is the objective function which is composed of buyer ordering cost, buyer inventory holding cost, vendor setup cost, and vendor inventory holding cost. Equation

(5.2) states that buyer i receives exactly m_i shipments. Equation (5.3) is a set of constraints that make sure each delivery is sent to only one buyer. The next set of constraints in equation 5.4 state that there has to be exactly one delivery to be sent to the ith buyer just after the jth replenishment. Constraints in equation (5.5) require x_{ik} to be one, if y_{ijk} is equal to one. Equation (5.6) is constraints that define the variable Y_{ijk} as function of y_{ijl}. Adding constraints in equation (5.7) to equation (5.3) guarantee that no delivery is sent to buyer i after the jth delivery. Constraints in equation 5.8 and equation 5.9 define T_{ij} and cycle length. Equation 5.10 defines T_{ij} in terms of j only i.e. the time required to deliver the next replenishment regardless of to whom is being delivered. Constraints in equation 5.11 are to enforce the amount produced by vendor for any buyer at any replenishment to be exactly equal to the amount needed to satisfy the demand during the time till next shipment received. Constraint in equation 5.12 is same as equation 5.11, but for last shipment, m. Equation 5.13 are non-negativity constraints.

5.1.2 Zero Switch Rule Based Heuristic

In zero switch rule (ZSR) the quantity produced by vendor to buyer i is equal to exactly the amount needed to satisfy the demand of buyer i till he/her receive the next shipment from the vendor. In this case we just need to know how many shipments should be sent to each buyer and in what order we have to order them to reduce the total cost. The remaining parameters will depend on these two variables. Hariga et al [20] provided a heuristic to solve the ZSR problem. Their heuristic procedure is based on two steps. First is to obtain the number of shipments that should to be sent to each buyer in a cycle. Second is to obtain the best sequencing of the orders using Bin Packing algorithm. For example, if we have two buyers: buyer1 and buyer2. We have to determine first how many shipments should be sent to buyer1 and buyer2. Then we choose the order to send to each buyer. For instance if we choose to send buyer 1 three shipments and buyer2 four shipment, we may order the buyers as 2 2 1 2 1 1 2. Then we obtain the quantities and the rest of the variables.

The heuristic procedure as in Hariga et al [20] states the following steps in order to solve the zero switch rule:

Step 1 : For each buyer i (*i=1* to *n*) determine an approximate delivery frequency using the following formula.

$$m_i = \sqrt{\frac{(h+h_i)D_iS}{S_iPh_i(1-\frac{D_i}{P})}}$$

$$m_i^* = 2^{round[\log_2(m_i)]}$$

Step 2: Generate a delivery sequence using a bin packing heuristic procedure.

Step 3: Find the timing of the shipments by solving a quadratic problem

Step 4: Compute the corresponding objective function value using (5.1)

The details of these steps are described in Hariga et al [20].

5.2 New Heuristic Solutions' Approach

In this section, we propose alternative heuristic solutions to the zero switch rule based formulation. Both heuristics make use of a procedure for finding a feasible solution to the ZSR problem given a sequence of shipments. We call this procedure "Near Feasible Solution Finder" and its details are presented in the next subsection. However, the proposed heuristics differ regarding the assumption on the number of shipments sent to each buyer as we will discuss later.

5.2.1 Near Feasible Solution Finder

Near Feasible Solution Finder (NFSF) is designed such that given any sequence and any starting value of t_j, the procedure will generate a feasible solution for the ZSR problem. The steps of this procedure are as follows:

Step 1: Set any starting values for all $t_j = \hat{t}_j$ for j = 1 to m. For example set all $\hat{t}_j$ to 0.001.

Note: It is preferable to take t_j in the range of 0.01 to 0.0001 depending on number of buyers and shipments to get closer feasible solutions to optimal solution.

Step 2: calculate $u_j = \begin{cases} t_{j+1}, j=1,2,....m-1 \\ t_1 + t_{j+1}, j=m \end{cases}$

Step 3: Calculate T_{ij} using (5.8)

Step 4: calculate $T_j = \sum_{i=1}^{n} T_{ij} x_{ij}$ for j=1 to m.

Step 5: Check if constraints (5.11)- (5.12) are feasible. If yes Stop

Step 6: Calculate $t_j^* = \dfrac{T_j \sum_{i=1}^{n} D_i x_{ij}}{P}$ for j = 1,2,3,….m

Step 7: Set $t_j = t_j^*$ and go to Step 2

Illustrative Example

S	400	i	S_i	D_i	h_i	m_i*
h	5	1	75	500	4	2
P	3200	2	25	1000	4	4
D	1500					

Given the sequence (1, 2, 2, 1, 2, 2) i.e the first shipment is sent to the first buyer, the next two shipments are sent to the second buyer, and so on.

Iteration#1

Step 1: take all t_j =0.01

$$u_j = \begin{cases} t_{j+1}, j=1,2,3,4,5 \\ t_1+t_7, j=6 \end{cases}$$

Step 2: Calculate $t_7 = (1-\frac{D}{P})T \ , T = \frac{P}{D}(t_1+t_2+t_3+t_4+t_5+t_6)$

Use the formula $T = \frac{P}{D}(t_1 + t_2 + t_3 + t_4 + t_5 + t_6)$ to calculate the cycle time T.

u_j	0.01	0.01	0.01	0.01	0.01	0.078

Step 3: For each i and j calculate $T_{ij} = u_j + \sum_{k=j=1}^{j-1}(1-Y_{ijk})u_k \quad i = 1,2$ and $j = 1,2,3,4,5,6$

Where $\quad Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$ and y_{ijk}=1 when the next delivery to the ith buyer after jth replenishment is made at the kth shipment and is zero otherwise.

	i	j	k=1	2	3	4	5	6
y_{ijk}	1	1	0	0	0	1	0	0
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	0	0	0	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	j	k=1	2	3	4	5	6
y_{ijk}	2	1	0	0	0	0	0	0
	2	2	0	0	1	0	0	0
	2	3	0	0	0	0	1	0
	2	4	0	0	0	0	0	0
	2	5	0	0	0	0	0	1
	2	6	0	1	0	0	0	0

	i	j	k=1	2	3	4	5	6
$Y_{ijk}=\sum_{l=i+1}^{k} y_{ijl}$	1	1	1	0	0	1	1	1
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	1	1	1	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	j	k=1	2	3	4	5	6
$Y_{ijk}=\sum_{l=i+1}^{k} y_{ijl}$	2	1	0	0	0	0	0	0
	2	2	1	1	1	1	1	1
	2	3	1	1	1	0	1	1
	2	4	0	0	0	0	0	0
	2	5	1	1	1	1	1	1
	2	6	0	1	1	1	1	1

T_{ij}	j	1	2	3	4	5	6
i	1	0.03	0.128	0.128	0.098	0.128	0.206
	2	0.128	0.01	0.02	0.128	0.01	0.088

Step 4: Calculate $T_j = \sum_{i=1}^{2} T_{ij} x_{ij}$ for j=1,2,…,6.

T_j*	0.03	0.01	0.02	0.098	0.01	0.088

Step 5: Equations (5.11) and (5.12) are not satisfied, so the solution is not feasible

Step 6 and 7: Calculate $t_j^* = \frac{T_j \sum_{i=1}^{n} D_i x_{ij}}{P}$ for j = 1,2,3,4,5,6 then set $t_j = t_j^*$ and go to step2

t_j	0.0046875	0.003125	0.00625	0.015313	0.003125	0.0275

$$u_j = \begin{cases} t_{j+1}, j=1,2,3,4,5 \\ t_1+t_7, j=6 \end{cases}$$

Step 2 : Calculate $t_7 = (1-\frac{D}{P})T \text{ },T = \frac{P}{D}(t_1+t_2+t_3+t_4+t_5+t_6)$

u_j	0.003125	0.00625	0.0153125	0.003125	0.0275	0.072688

Step 3: For each i and j calculate $T_{ij} = u_j + \sum_{k=j=1}^{j-1}(1-Y_{ijk})u_k \quad i=1,2$ and $j=1,2,3,4,5,6$

Where $Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$ and y_{ijk}=1 when the next delivery to the ith buyer after jth replenishment is made at the kth shipment and is zero otherwise.

	i	*j*	*k*=1	2	3	4	5	6
y_{ijk}	1	1	0	0	0	1	0	0
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	0	0	0	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	*j*	*k*=1	2	3	4	5	6
y_{ijk}	2	1	0	0	0	0	0	0
	2	2	0	0	1	0	0	0
	2	3	0	0	0	0	1	0
	2	4	0	0	0	0	0	0
	2	5	0	0	0	0	0	1
	2	6	0	1	0	0	0	0

	i	*j*	*k*=1	2	3	4	5	6
$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	1	1	1	0	0	1	1	1
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	1	1	1	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	*i*	*j*	*k*=1	2	3	4	5	6
	2	1	0	0	0	0	0	0
	2	2	1	1	1	1	1	1
	2	3	1	1	1	0	1	1
	2	4	0	0	0	0	0	0
	2	5	1	1	1	1	1	1
	2	6	0	1	1	1	1	1

T_{ij}	*j*	1	2	3	4	5	6
i	1	0.024688	0.128	0.128	0.103313	0.128	0.200688
	2	0.128	0.00625	0.018438	0.128	0.0275	0.075813

Step 4: Calculate $T_j = \sum_{i=1}^{2} T_{ij} x_{ij}$ for j=1,2,3,4,5,6.

x_{ij} =1 if the jth shipment is sent to *ith* buyer and 0 otherwise.

xij	j	1	2	3	4	5	6
i	1	1	0	0	1	0	0
	2	0	1	1	0	1	1

T_j*	0.0246875	0.00625	0.0184375	0.103313	0.0275	0.075813

Step 5: Equations (5.11) and (5.12) are not satisfied, so the solution is not feasible

Step 6 and 7: Calculate $t_j^* = \dfrac{T_j \sum_{i=1}^{n} D_i x_{ij}}{P}$ for j = 1,2,3,4,5,6 then set $t_j = t_j^*$ and go to step2

t_j*	0.003857422	0.00195313	0.005761719	0.016143	0.008594	0.023691

Iteration#2

$$u_j = \begin{cases} t_{j+1}, j = 1,2,3,4,5 \\ t_1 + t_7, j = 6 \end{cases}$$

Step 2: Calculate $$t_7 = (1 - \frac{D}{P})T, T = \frac{P}{D}(t_1 + t_2 + t_3 + t_4 + t_5 + t_6)$$

U_j	0.001953125	0.00576172	0.016142578	0.008594	0.023691	0.071857

Step 3: For each i and j calculate $T_{ij} = u_j + \sum_{k=j=1}^{j-1} (1 - Y_{ijk}) u_k \quad i = 1,2$ and $j = 1,2,3,4,5,6$

Where $\quad Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$

	i	*j*	*k*=1	2	3	4	5	6
y_{ijk}	1	1	0	0	0	1	0	0
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	0	0	0	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	*j*	*k*=1	2	3	4	5	6
y_{ijk}	2	1	0	0	0	0	0	0
	2	2	0	0	1	0	0	0
	2	3	0	0	0	0	1	0
	2	4	0	0	0	0	0	0
	2	5	0	0	0	0	0	1
	2	6	0	1	0	0	0	0

	i	*j*	*k*=1	2	3	4	5	6
$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	1	1	1	0	0	1	1	1
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	1	1	1	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	*j*	*k*=1	2	3	4	5	6
$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	2	1	0	0	0	0	0	0
	2	2	1	1	1	1	1	1
	2	3	1	1	1	0	1	1
	2	4	0	0	0	0	0	0
	2	5	1	1	1	1	1	1
	2	6	0	1	1	1	1	1

T_{ij}	j	1	2	3	4	5	6
i	1	0.023857	0.128	0.128	0.104143	0.128	0.199857
	2	0.128	0.005762	0.024736	0.128	0.023691	0.073811

Step 4: Calculate $T_j = \sum_{i=1}^{2} T_{ij} x_{ij}$ for j=1,2,3,4,5,6.

T_j^*	0.107269879	0.02590636	0.111221696	0.468255	0.106523	0.331874

Step 5: Equations (5.11) and (5.12) are not satisfied, so the solution is not feasible

Step 6 and 7: Calculate $t_j^* = \frac{T_j \sum_{i=1}^{n} D_i x_{ij}}{P}$ for j = 1,2,3,4,5,6 then set $t_j = t_j^*$ and go to step2

$t_jT_j^*$	0.003727722	0.00180054	0.007730103	0.016272	0.007404	0.023066

Iteration#3

$$u_j = \begin{cases} t_{j+1}, j = 1,2,3,4,5 \\ t_1 + t_7, j = 6 \end{cases}$$

Step 2: Calculate $$t_7 = (1 - \frac{D}{P})T, T = \frac{P}{D}(t_1 + t_2 + t_3 + t_4 + t_5 + t_6)$$

u_j	0.001800537	0.0077301	0.016272278	0.007404	0.023066	0.071728

Step 3: For each i and j calculate $T_{ij} = u_j + \sum_{k=j=1}^{j-1} (1 - Y_{ijk}) u_k \quad i = 1,2$ and $j = 1,2,3,4,5,6$

Where $Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$

	i	j	k=1	2	3	4	5	6
y_{ijk}	1	1	0	0	0	1	0	0
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	0	0	0	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	j	k=1	2	3	4	5	6
y_{ijk}	2	1	0	0	0	0	0	0
	2	2	0	0	1	0	0	0
	2	3	0	0	0	0	1	0
	2	4	0	0	0	0	0	0
	2	5	0	0	0	0	0	1
	2	6	0	1	0	0	0	0

	i	j	k=1	2	3	4	5	6
$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	1	1	1	0	0	1	1	1
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	1	1	1	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	j	k=1	2	3	4	5	6
$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	2	1	0	0	0	0	0	0
	2	2	1	1	1	1	1	1
	2	3	1	1	1	0	1	1
	2	4	0	0	0	0	0	0
	2	5	1	1	1	1	1	1
	2	6	0	1	1	1	1	1

T_{ij}	j	1	2	3	4	5	6
i	1	0.025803	0.128	0.128	0.102197	0.128	0.199728
	2	0.128	0.00773	0.023676	0.128	0.023066	0.073528

Step 4: Calculate $T_j = \sum_{i=1}^{2} T_{ij} x_{ij}$ for j=1,2,3,4,5,6.

T_j*	0.025802917	0.0077301	0.023675842	0.102197	0.023066	0.073528

Step 5: Equations (5.11) and (5.12) are not satisfied, so the solution is not feasible

Step 6 and 7: Calculate $t_j^* = \dfrac{T_j \sum_{i=1}^{n} D_i x_{ij}}{P}$ for j = 1,2,3,4,5,6 then set $t_j = t_j^*$ and go to step2

t_jT_j*	0.004031706	0.00241566	0.007398701	0.015968	0.007208	0.022978

Iteration#4

Step 2:

u_j	0.002415657	0.0073987	0.015968294	0.007208	0.022978	0.072032

Step 3: For each i and j calculate $T_{ij} = u_j + \sum_{k=j=1}^{j-1} (1 - Y_{ijk}) u_k \quad i = 1,2$ and $j = 1,2,3,4,5,6$

Where $Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$

	i	j	k=1	2	3	4	5	6
y_{ijk}	1	1	0	0	0	1	0	0
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	0	0	0	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	j	k=1	2	3	4	5	6
y_{ijk}	2	1	0	0	0	0	0	0
	2	2	0	0	1	0	0	0
	2	3	0	0	0	0	1	0
	2	4	0	0	0	0	0	0
	2	5	0	0	0	0	0	1
	2	6	0	1	0	0	0	0

	i	j	k=1	2	3	4	5	6
$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	1	1	1	0	0	1	1	1
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	1	1	1	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	j	k=1	2	3	4	5	6
$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	2	1	0	0	0	0	0	0
	2	2	1	1	1	1	1	1
	2	3	1	1	1	0	1	1
	2	4	0	0	0	0	0	0
	2	5	1	1	1	1	1	1
	2	6	0	1	1	1	1	1

T_{ij}	j	1	2	3	4	5	6
i	1	0.025783	0.128	0.128	0.102217	0.128	0.200032
	2	0.128	0.007399	0.023176	0.128	0.022978	0.074447

Step 4: Calculate $T_j = \sum_{i=1}^{2} T_{ij} x_{ij}$ for j=1,2,3,4,5,6.

T_j*	0.116541706	0.03344331	0.10476083	0.462039	0.103862	0.336514

Step 5: Equations (5.11) and (5.12) are not satisfied, so the solution is not feasible

Step 6 and 7: Calculate $t_j^* = \dfrac{T_j \sum_{i=1}^{n} D_i x_{ij}}{P}$ for j = 1,2,3,4,5,6 then set $t_j = t_j^*$ and go to step2

$t_j T_j$*	0.004028539	0.00231209	0.007242611	0.015971	0.00718	0.023265

Iteration#5

$$u_j = \begin{cases} t_{j+1}, j = 1,2,3,4,5 \\ t_1 + t_7, j = 6 \end{cases}$$

Step 2: Calculate
$$t_7 = (1 - \frac{D}{P})T, T = \frac{P}{D}(t_1 + t_2 + t_3 + t_4 + t_5 + t_6)$$

u_j	0.002312094	0.00724261	0.015971461	0.00718	0.023265	0.072029

Step 3: For each *i* and *j* calculate $T_{ij} = u_j + \sum_{k=j=1}^{j-1} (1 - Y_{ijk}) u_k \quad i = 1, 2$ and $j = 1, 2, 3, 4, 5, 6$

Where $Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$

y_{ijk}	*i*	*j*	*k*=1	2	3	4	5	6
	1	1	0	0	0	1	0	0
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	0	0	0	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

y_{ijk}	*i*	*j*	*k*=1	2	3	4	5	6
	2	1	0	0	0	0	0	0
	2	2	0	0	1	0	0	0
	2	3	0	0	0	0	1	0
	2	4	0	0	0	0	0	0
	2	5	0	0	0	0	0	1
	2	6	0	1	0	0	0	0

$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	*i*	*j*	*k*=1	2	3	4	5	6
	1	1	1	0	0	1	1	1
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	1	1	1	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	*i*	*j*	*k*=1	2	3	4	5	6
	2	1	0	0	0	0	0	0
	2	2	1	1	1	1	1	1
	2	3	1	1	1	0	1	1
	2	4	0	0	0	0	0	0
	2	5	1	1	1	1	1	1
	2	6	0	1	1	1	1	1

T_{ij}	*j*	1	2	3	4	5	6
i	1	0.025526	0.128	0.128	0.102474	0.128	0.200029
	2	0.128	0.007243	0.023152	0.128	0.023265	0.074341

Step 4: Calculate $T_j = \sum_{i=1}^{2} T_{ij} x_{ij}$ for j=1,2,3,4,5,6.

T_j*	0.025526166	0.00724261	0.023151955	0.102474	0.023265	0.074341

Step 5: Equations (5.11) and (5.12) are not satisfied, so the solution is not feasible

Step 6 and 7: Calculate $t_j^* = \frac{T_j \sum_{i=1}^{n} D_i x_{ij}}{P}$ for j = 1,2,3,4,5,6 then set $t_j = t_j^*$ and go to step2

$t_j T_j$*	0.003988391	0.00226332	0.007234688	0.016011	0.00727	0.023231

Iteration#6

$$u_j = \begin{cases} t_{j+1}, j = 1,2,3,4,5 \\ t_1 + t_7, j = 6 \end{cases}$$

Step 2: Calculate $t_7 = (1 - \frac{D}{P})T, T = \frac{P}{D}(t_1 + t_2 + t_3 + t_4 + t_5 + t_6)$

u_j	0.002263316	0.00723499	0.016011537	0.00727	0.023231	0.071988

Step 3: For each i and j calculate $T_{ij} = u_j + \sum_{k=j=1}^{j-1} (1 - Y_{ijk}) u_k$ $i = 1,2$ and $j = 1,2,3,4,5,6$

Where $Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$

	i	j	k=1	2	3	4	5	6
y_{ijk}	1	1	0	0	0	1	0	0
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	0	0	0	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	j	k=1	2	3	4	5	6
y_{ijk}	2	1	0	0	0	0	0	0
	2	2	0	0	1	0	0	0
	2	3	0	0	0	0	1	0
	2	4	0	0	0	0	0	0
	2	5	0	0	0	0	0	1
	2	6	0	1	0	0	0	0

$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	*i*	*j*	*k*=1	2	3	4	5	6
	1	1	1	0	0	1	1	1
	1	2	0	0	0	0	0	0
	1	3	0	0	0	0	0	0
	1	4	1	1	1	1	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	*i*	*j*	*k*=1	2	3	4	5	6
	2	1	0	0	0	0	0	0
	2	2	1	1	1	1	1	1
	2	3	1	1	1	0	1	1
	2	4	0	0	0	0	0	0
	2	5	1	1	1	1	1	1
	2	6	0	1	1	1	1	1

T_{ij}	*j*	1	2	3	4	5	6
i	1	0.02551	0.128	0.128	0.10249	0.128	0.199988
	2	0.128	0.007235	0.023282	0.128	0.023231	0.074252

Step 4: Calculate $T_j = \sum_{i=1}^{2} T_{ij} x_{ij}$ for j=1,2,3,4,5,6.

T_j*	0.025509838	0.00723499	0.023281787	0.10249	0.023231	0.074252

Step 5: Equations (5.11) and (5.12) are not satisfied, so the solution is not feasible

Step 6 and 7: Calculate $t_j^* = \dfrac{T_j \sum_{i=1}^{n} D_i x_{ij}}{P}$ for j = 1,2,3,4,5,6 then set $t_j = t_j^*$ and go to step2

t_jT_j*	0.003985837	0.00226084	0.007275521	0.016014	0.00726	0.023203

Iteration#7

$$u_j = \begin{cases} t_{j+1}, j=1,2,3,4,5 \\ t_1 + t_7, j=6 \end{cases}$$

Step 2: Calculate $$t_7 = (1 - \frac{D}{P})T, T = \frac{P}{D}(t_1 + t_2 + t_3 + t_4 + t_5 + t_6)$$

u_j	0.002260933	0.00727556	0.016014088	0.00726	0.023204	0.071986

Step 3: For each i and j calculate $T_{ij} = u_j + \sum_{k=j=1}^{j-1} (1 - Y_{ijk}) u_k \quad i = 1,2$ and $j = 1,2,3,4,5,6$

Where $Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$

	i	j	k=1	2	3	4	5	6
	1	1	0	0	0	1	0	0
	1	2	0	0	0	0	0	0
y_{ijk}	1	3	0	0	0	0	0	0
	1	4	1	0	0	0	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	j	k=1	2	3	4	5	6
	2	1	0	0	0	0	0	0
	2	2	0	0	1	0	0	0
y_{ijk}	2	3	0	0	0	0	1	0
	2	4	0	0	0	0	0	0
	2	5	0	0	0	0	0	1
	2	6	0	1	0	0	0	0

	i	j	k=1	2	3	4	5	6
	1	1	1	0	0	1	1	1
	1	2	0	0	0	0	0	0
$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	1	3	0	0	0	0	0	0
	1	4	1	1	1	1	0	0
	1	5	0	0	0	0	0	0
	1	6	0	0	0	0	0	0

	i	j	k=1	2	3	4	5	6
	2	1	0	0	0	0	0	0
	2	2	1	1	1	1	1	1
$Y_{ijk} = \sum_{l=i+1}^{k} y_{ijl}$	2	3	1	1	1	0	1	1
	2	4	0	0	0	0	0	0
	2	5	1	1	1	1	1	1
	2	6	0	1	1	1	1	1

T_{ij}	j	1	2	3	4	5	6
i	1	0.02555	0.127998	0.127998	0.102448	0.127998	0.199983
	2	0.127998	0.007276	0.023274	0.127998	0.023203	0.074246

Step 4: Calculate $T_j = \sum_{i=1}^{2} T_{ij} x_{ij}$ for j=1,2,3,4,5,6.

T_j*	0.025509838	0.00723499	0.023281787	0.10249	0.023231	0.074252

Step 5: Equations (5.11) and (5.12) are satisfied, so the solution is feasible

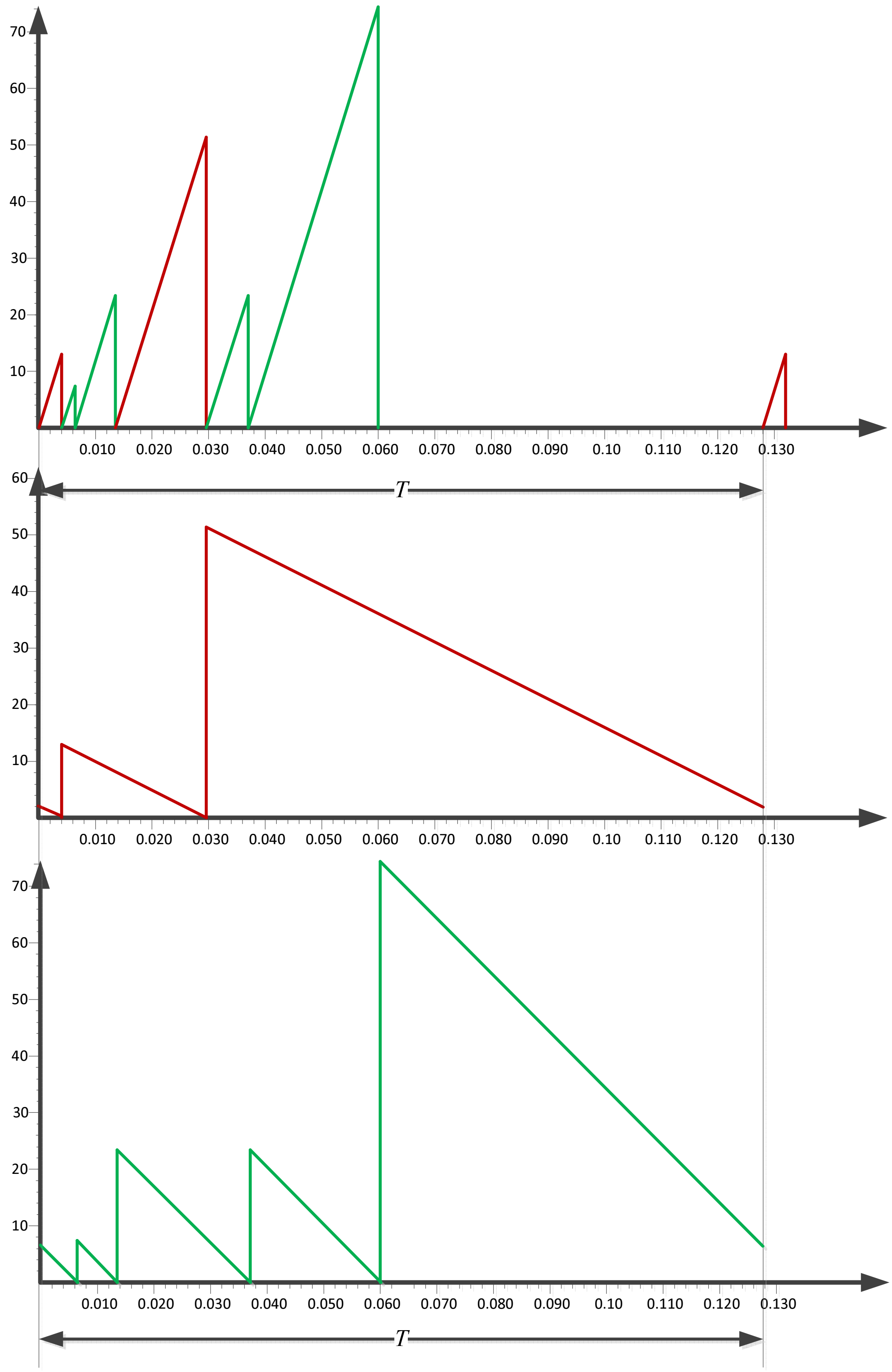

Figure 5.3 Solution generated by NFSF for a 2-buyer problem**.**

5.2.2 First Heuristic

The first heuristic is based on a fixed number of shipments determined by assuming that the vendor is dealing with only one buyer and determining the corresponding optimal number of shipment as mentioned in Section 5.1.1. It can be shown that such optimal number is given by (Hariga et al [20]).

$$m_i = \sqrt{\frac{(h+h_i)D_iS}{S_iPh_i(1-\frac{D_i}{P})}} \qquad (5.14)$$
$$m_i^* = 2^{round[\log_2(m_i)]}$$

In this first heuristic, we present an alternative method to the bin packing heuristic proposed in Hariga et al [20] to obtain the best sequence of shipments given a fixed number of shipments to each buyer determined by equations (5.14).

To implement our heuristic the following algorithm is used.

Step 1: Given any sequence, run NFSF algorithm to find a feasible solution to the ZSR problem (5.1)-(5.13). It is better to use t_j in the range of 0.01 to 0.0001.

Step 2: Relax constraints (5.11) & (5.12) and use genetic algorithm to change the sequence by using reordering method. We used Evolver software to run the genetic algorithm to obtain the best solution to the relaxed problem. The purpose of this step is to generate a better sequence and is an alternative to the bin packing heuristic proposed by Hariga et al. [20].

Note: In the case of small number of buyers like 2 buyers, you may not use step 2 but use manual swapping until you reach the minimum of the relaxed problem.

Step 3: If a lower relaxed objective value is obtained in Step 2, then go to Step 1. If this is not the case, stop since the best solution so far cannot be improved.

Figure 5.3 shows a flow chart for this algorithm.

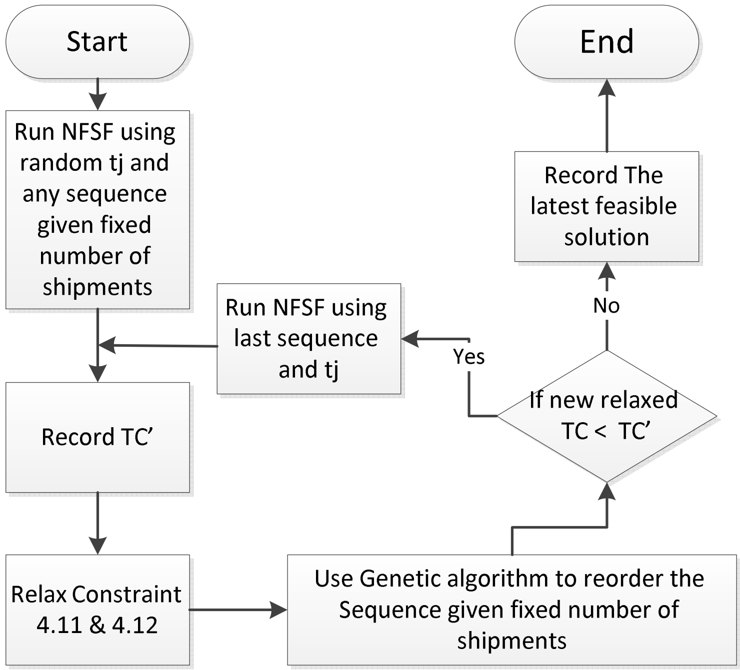

Figure 5.4: First heuristic algorithms

5.2.3 Second Heuristic

In the second heuristic, both the sequence and number of shipments were changed in order to obtain better solutions. In this case, the number of shipments to each buyer is changed. However, we assume that the total number of shipments for all buyers $m = \sum_{i=1}^{n} m_i$ is equal to total number of shipments given by equation (5.14)

Table 5.1 shows the differences between the two heuristics and special assumptions.

Table 5.1: New heuristics' approach

	Sequence	Number of Shipments
Hariga [20] ZSR heuristic	Bin packing heuristic	Fixed
First heuristic	Generic algorithm	Fixed
Second heuristic	Generic algorithm	Variable but the total for all buyer if fixed

5.3 Illustrative Examples to Heuristics

In order to illustrate the two heuristics in this chapter we may take one illustrative example. Assume we have a five buyer's problem with the following data as in table 5.2:

Table 5.2: Vendor & 5 buyers data

Vendor data				
S	400			
H	10			
P	7000			
D	1544			
Buyers data				
I	Si	Di	hi	mi*
1	58	734	2	2
2	37	360	3	2
3	20	200	1	2
4	23	100	3	1
5	25	150	2	2

As in the data table the column m^*_i is the number of shipments given (5.14).

Based on (5.14) as in table 5.2, a total of 9 shipments were generated and if we run the

Bin packing algorithm the optimal sequence according to it will be:

2	1	5	3	4	2	1	5	3

Solving the problem lead to a total cost of 2258.869874 with the following timings of shipments t_j:

J	1	2	3	4	5	6	7	8	9
t_j	0.002260991	0.00840326	0.001869198	0.002748	0.007218	0.023725	0.04458	0.008958365	0.011689

5.3.1 First Heuristic Implementation

Step1: take t_j = 0.02 for j=1 to 9 and choose a random sequence based (5.14) number of shipments.

j	1	2	3	4	5	6	7	8	9
i	1	2	3	4	5	1	2	3	5

Now run NFSF we get TC= 2254.6225984310

Step 2: relax (5.11) & (5.12) and maintain all other constraints. Record the last TC and fix the timings.

Step 3: run genetic algorithm to reorder the sequence till the minimum relax TC. We get Relaxed TC= 2151.9830272692. Based on the sequence:

1	2	5	3	4	3	5	2	1

Step 4: Since Relax TC < original TC go to step 1 and run NFSF. We get TC= 2242.1312571926

Now in step 2 relax constraints (5.11) & (5.12).

In step 3 by reordering using genetic, we find no more improved solution so we stop here. The best solution we get is better than the solution obtained by the heuristic in Hariga [20].

j	1	2	3	4	5	6	7	8	9
Seq. i	1	2	5	3	4	3	5	2	1
t_j	0.01708301	0.00461886	0.001100583	0.000972	0.011658	0.022345	0.016386	0.037350051	0.068487
TC	2242.1312571926								

5.3.2 Second Heuristic Implementation

Step1: take t_j = 0.02 for j=1 to 9 and choose a random sequence based on total fixed number of shipments which is 9. So the total number of shipments m=9.

j	1	2	3	4	5	6	7	8	9
i	1	4	3	4	4	1	2	3	5

Now run NFSF we get TC= 2421.2488130665

Step 2: relax (5.11) & (5.12) and maintain all other constraints. Record the last TC and fix the timings.

Step 3: run genetic algorithm to reorder the sequence based on grouping method till the minimum relax TC. We get Relaxed TC= 2149.314906. Based on the sequence:

2	3	5	1	4	3	5	2	1

Step 4: Since Relax TC < original TC go to step 1 and run NFSF. We get TC= 2229.357897

Now in step 2 relax constraints (5.11) & (5.12) and continue on steps. Table 5.3 will show the reaming iterations

Table 5.3: The second heuristic iterations

iteration	j=	1	2	3	4	5	6	7	8	9
2	Sequence i	1	3	2	4	5	3	4	2	1
	t_j	0.01708301	0.00128	0.004556	0.000721	0.017487	0.022036056	0.010937	0.03741266	0.068487
	Relaxed TC		2209.545608		Original TC		2239.482591			
3	Sequence i	1	2	3	4	5	3	4	2	1
	t_j	0.01708301	0.004619	0.001153	0.000723	0.017487	0.022162569	0.010935	0.037350051	0.068487
	Relaxed TC		2236.051552		Original TC		2239.264552			

In iteration 3 we stop where no more improvements. Among all iterations, iteration 1 have the minimum cost so the best solution to the problem is in the following table 5.4 and the obtained results is better than the solution obtained by the heuristic in Hariga [20] and in the first heuristic.

Table 5.4: The best solution by second heuristic

j	1	2	3	4	5	6	7	8	9
Seq. i	2	3	5	1	4	3	5	2	1
t_j	0.009630354	6.3E-05	0.004245	0.000163	0.011432	0.075939594	0.041969	0.019071073	0.017487
TC	2229.357897								

5.4 Heuristics implementation and Numerical Comparisons

In this section, we first regenerated the 19 problems of two buyers solved in the reviewed paper. Then we generated a larger number of buyers (up to 30 buyers). The problems were first solved using the heuristic in the paper and later we had solved them using our two heuristics.

Table 5.1 &5.2 will show the 19 problems of two buyers regenerated and solved using the both heuristics. Table 5.3 & 5.4 & 5.5 will show the larger problems where we had 10 buyers and 10, 15, 20, 25, 30 buyer's problems.

Table 5.5: 19 problems of two buyers solution part 1

pro. #	heuristic solution in paper [20]		regenerated solution and optimized based on heuristic in paper [20]					optimizing sequence solution with fixed frequency				
	cost solution	Frequency	cost solution	improvement%	sequence	# of trials	mean time to get solution	cost solution	improvement%	sequence	# of trials	mean time to get solution
1	1656.62	2,2	1668.108	-0.6935%	2121	2	00:43	1656.65	0.6869%	1212	3	00:39
2	1356.29	1,4	1356.312	-0.0016%	21222	1	00:52	1349.461	0.5051%	22122	2	00:48
3	2138.27	2,4	2138.269	0.0000%	21221	1	00:56	-	-	-	-	-
4	1420.98	2,4	1420.497	0.0340%	212212	2	00:56	1419.595	0.0635%	212221	2	00:46
5	2141.29	2,4	2140.792	0.0232%	212212	2	01:02	-	-	-	-	-
6	1644.84	2,4	1644.843	-0.0002%	212212	1	00:42	-	-	-	-	-
7	1357.14	8,2	1357.165	-0.0018%	1211112111	2	01:30	1350.386	0.4995%	1111211112	1	01:10
8	1290.29	4,2	1290.29	0.0000%	121121	1	00:55	1230.423	4.6398%	121112	1	00:52
9	1414.6	4,2	1414.628	-0.0020%	121121	1	00:52	-	-	-	-	-
10	1466.32	2,2	1448.877	1.1896%	1212	1	00:31	1429.996	1.3031%	2121	1	00:39
11	1146.91	2,2	1146.215	0.0606%	1212	1	00:32	-	-	-	-	-
12	1804.86	4,4	1842.689	-2.0959%	21212121	1	00:56	1804.891	2.0513%	12121212	1	00:49
13	1844.7	4,2	1844.697	0.0002%	121121	1	00:57	-	-	-	-	-
14	1276.2	1,2	1276.205	-0.0004%	212	1	00:31	-	-	-	-	-
15	1409.55	2,4	1409.701	-0.0107%	212212	1	00:43	1397.614	0.8574%	212221	1	00:41
16	1281.81	2,2	1303.336	-1.6793%	2121	1	00:36	1281.823	1.6506%	1212	1	00:30
17	1232.19	2,4	1231.805	0.0313%	212212	1	00:54	-	-	-	-	-
18	1860.78	4,4	1861.003	-0.0120%	21212121	1	01:05	1860.383	0.0333%	12121212	1	01:13
19	1509.19	2,8	1509.708	-0.0343%	2122221222	1	01:15	1502.275	0.4923%	2221222212	1	01:20

Table 5.6: 19 problems of two buyers solution part 2

pro. #	optimizing #of shipments and sequence					
	solution cost	improvement%	frequency	sequence	# of trials	mean time to get solution
1	-	-	-	-	-	-
2	1343.963	0.9105%	2,3	21221	2	00:56
3	2030.199	5.0541%	3,3	21212	1	00:50
4	1377.974	2.9935%	3,3	212121	2	00:50
5	2047.468	4.3593%	3,3	212121	1	00:49
6	1623.288	1.3105%	3,3	121212	1	00:59
7	1341.244	1.1731%	6,4	1211211212	1	01:21
8	1229.112	4.7414%	3,3	121212	1	01:03
9	-	-	-	-	-	-
10	-	-	-	-	-	-
11	-	-	-	-	-	-
12	1798.401	2.4034%	3,5	21212122	1	00:52
13	1752.508	4.9975%	2,4	212212	1	00:49
14	-	-	-	-	-	-
15	1347.702	4.3980%	4,2	121211	1	00:40
16	-	-	-	-	-	-
17	-	-	-	-	-	-
18	-	-	-	-	-	-
19	1432.472	5.1159%	5,5	2121212121	1	01:12

Table 5.7: Larger problems solution part 1

pro. #	data	the solution based on heuristic in paper [20]	optimizing sequence solution with fixed frequency					optimizing #of shipments and sequence				
	number of buyers	solution cost	solution cost	Optimized sequence	improvement %	# of trials	mean time to get solution	solution cost	optimized frequency & sequence	improvement%	# of trials	mean time to get solution
1	10	3439.4825	3399.0227	2 9 7 6 3 8 1 10 5 4 8 3 5 7 10 2 9 6 1	1.18%	3	0:39:00	3353.506	9 2 7 6 5 4 1 8 10 9 3 8 5 6 7 4 9 2 1	2.50%	2	0:40:00
2	10	4005.3027	3914.02234	5 8 6 9 2 7 4 8 1 5 10 3 8 5 9 4 2 6 7 8 5	2.28%	2	0:48:00	3610.254	2 5 6 3 10 7 1 8 4 5 9 6 5 2 3 1 8 6 7 10 5	9.86%	2	2:15:00
3	10	4006.6749	3932.62985	6 5 8 9 7 4 10 6 5 2 3 1 5 6 9 8 5 7 6 4 10	1.85%	3	1:30:00	3912.656	6 5 8 9 7 4 10 6 7 2 3 1 7 6 9 8 5 7 6 4 10	2.35%	2	1:25:00
4	10	4114.9499	4066.09065	8 6 2 10 7 5 8 9 1 3 8 4 7 5 2 1 8 6 10 3 9	1.19%	3	1:07:00	4018.449	8 6 2 10 1 5 3 9 1 4 8 6 7 5 2 1 8 6 10 3 9	2.35%	2	1:04:30
5	10	4656.797	4525.14273	7 2 1 3 8 9 4 5 6 10 9 8 2 7 1 3	2.83%	3	1:10:00	4399.324	3 7 1 2 6 9 8 5 10 1 4 2 7 3 6 1	5.53%	2	3:00:00

Table 5.8: Larger problems solution part 2

pro. #	data	the solution based on heuristic in paper [20]	optimizing sequence solution with fixed frequency					optimizing #of shipments and sequence				
	number of buyers	solution cost	solution cost	Optimized sequence	improvement %	# of trials	mean time to get solution	solution cost	optimized frequency & sequence	improvement%	# of trials	mean time to get solution
6	10	4015.1204	3905.4753	8 7 5 10 6 2 4 1 8 7 3 9 8 4 7 6 5 8 10 1 2 7	2.73%	3	0:45:00	3801.645	9 7 5 10 6 2 4 1 8 7 3 10 8 4 7 6 5 9 1 10 2 7	5.32%	2	0:38:00
7	10	4143.4556	4063.40921	2 1 8 5 6 4 10 7 8 9 3 8 8 2 1 6 4 5 10 7	1.93%	1	1:42:00	3971.074	2 7 5 1 6 4 10 7 8 9 3 5 8 2 1 6 4 5 10 7	4.16%		1:12:00
8	10	4446.5953	4350.27891	6 1 8 3 10 9 4 5 6 1 2 7 1 6 9 10 1 6 8 3 4 5	2.17%	4	0:54:00	4096.198	4 1 3 8 10 2 6 5 9 7 4 1 9 3 6 10 2 8 5 4 3 7	7.88%	4	0:42:00
9	10	3909.0781	3893.71206	6 10 3 9 1 4 8 2 5 6 9 7 9 6 8 1 4 9 10 3 6 2 5	0.39%	2	0:56:00	3788.019	6 10 3 2 1 4 8 9 5 6 2 7 10 2 8 1 4 9 10 3 6 2 5	3.10%	2	0:30:00
10	10	4467.6835	4390.57911	8 5 3 7 10 2 4 8 5 3 1 6 9 3 8 5 7 10 3 2 4 8 5	1.73%	1	1:05:00	4191.011	9 6 8 10 7 2 4 5 1 3 8 10 4 5 8 9 7 10 6 2 4 8 5	6.19%	2	1:41:00

Table 5.9: Larger problem solution part 3

pro. #	data	the solution based on heuristic in paper [20]	optimizing sequence solution with fixed frequency					optimizing #of shipments and sequence				
	number of buyers	solution cost	solution cost	Optimized sequence	improvement%	# of trials	mean time to get solution	solution cost	optimized frequency & sequence	improvement%	# of trials	mean time to get solution
11	15	6659.0186	6480.26809	12 3 11 1 9 10 8 15 4 2 5 6 14 7 13 12 3 1 11 8 9 10 15 4	2.68%	4	1:34:00	6340.088	12 11 13 1 9 10 3 15 4 5 2 6 8 7 13 3 12 11 14 5 9 10 15 4	4.79%	2	1:25:00
12	20	8068.9275	8005.43088	3 1 18 13 14 20 12 15 8 17 19 4 7 9 6 16 11 5 10 2 13 1 3 18 14	0.79%	4	1:05:00	7770.741	3 4 18 9 13 7 1 11 8 17 19 20 15 9 6 16 12 5 10 7 2 14 3 18 4	3.70%	1	1:55:00
13	25	8729.9493	8575.33554	15 17 12 13 18 2 16 22 11 8 6 14 3 25 10 21 19 1 23 5 9 7 24 20 18 2 15 4 12 17 13 14 22	1.77%	6	2:25:00	8383.984	15 17 12 13 18 4 16 25 23 8 6 14 3 22 10 21 19 11 1 5 9 7 24 20 18 2 15 4 12 17 13 23 25	3.96%	3	1:19:00
14	30	11089.314	11029.8912	15 23 6 21 3 25 14 30 8 9 17 20 10 7 12 26 11 22 2 13 27 5 28 4 19 1 15 23 16 3 18 24 14 30 29	0.54%	5	3:05:00	10608.55	6 23 15 21 3 28 14 4 29 26 17 2 8 7 12 9 11 19 24 13 27 30 25 10 22 5 1 23 16 17 18 2 26 20 29	4.34%	4	2:45:00

In the 19 problems of two buyers, most of problems (15 out of 19) we obtained better results with improvements of 4.6% percent. The result is a good indication that Bin Packing algorithm will not provide better solutions. Relaxing the number of shipments in second heuristic, lead to improvements up to 5.12%, which indicated that Zavanell's [8] number of shipments, was not giving the best solution to the model.

In larger cases, all problems were improved by both the above stated heuristics. In the first heuristic, the improvements of 2.8% were recorded and that emphasized the statement that Bin Packing was not an appropriate way to get better results. Using the second heuristic, the improvements of around 10% were recorded and this indicated that fixed number of shipments was not the best choice to have better results.

CHAPTER 6

Summary & Conclusions

This chapter provides summary and conclusion of the thesis work, highlight some of the interesting results we have achieved and suggest some future researches and extensions.

6.1 Research Results & Conclusions

In this thesis, we intensively studied the theory of “single vendor multi buyer” problems. We reviewed the relevant recent models in literature and discovered their weaknesses. We succeeded in relaxing and extending the state-of-the-art models, which resulted in better results. The extended models were more efficient and indicated that various models in literature assumed things, which were not beneficial to the model. Later we designed two heuristics to solve the general model and found better solutions as compared to those existing in literature. Below is the summary of the contribution of this work.

- Hoque in [16 & 18] restricted his models to the policy that the vendor sent his/ her shipments to all buyers at the same time and each buyer took the proportional amount to his/her demand for the big shipment. This caused Hoque model to become expensive because the vendor would have to store huge amount of inventory in his/her stock. Instead, he could send to each buyer alone so that the buyer could optimize his lot according to his /her demand. Therefore, we developed two models with cyclic shipments to each buyer. We found out that our new model would incur less cost and improvement would be very high as the number of buyer's increases.

- Bendaya, et al in [6] implemented some strategies to manage the inventory between vendor and buyers. However, their model was not optimal to supply chain relation. They assumed that for cyclic shipments, the vendor sent the same size of shipments to each buyer although not the same size for different buyers. This made their model to charge the vendor high amount of inventory and increase in number of shipments cost. We designed an extended model that let the vendor send different size of shipments to each buyer and improved their model. In one such case, we saw an improvement of 10.98% in cost with the help of our model.
- Hariga et al in [20] proposed the most general "single vendor multi buyers" supply chain model. They proposed a heuristic to solve their model under a condition of "zero switch role". The proposed heuristic in the paper did not give good results. We designed two heuristics to get better results than the heurists given in the paper under the same condition of "zero switch rule". The first heuristic relaxed the use of bin packing algorithm and the second relaxed the fixed number of shipments to each buyer. As a result, we recorded better results for most of the small problems of two buyers. However, in cases where there were up to 30 buyers, we found 10% improvement in all cases. In addition, the time to get these solutions was lesser.

6.2 Future Studies

For future studies, we suggest to focus on the general model of Hariga in [20] and tried to get better ways to get better and faster solutions. In the last heuristic, we assumed that the total number of shipments 'm' was equal to the total number of fixed number of shipments and this assumption could be relaxed and hence result in better results.

Another point of consideration would be to work on the general problem without zero switch roles to get some fast and efficient heuristics to solve the general formulation and will help more to get good results. To simplify things one may use our heuristics to start with some zero switch solution and try to optimize the general model using that as the starting solution.

We may think about some general models that have the policy that the shipments could not be sent to buyers until the buyers consume his/her lot completely. That is the accumulation of inventory would be with vendor and not the buyer and hence the buyers do not need to have any inventory.

References

[1]. Abdul-Jalbar, B., Gutierrez, J.M., Sicilia, J., "Single cycle policies for the one-warehouse N-retailer inventory/distribution system." *Omega 34* ed. 2, 2006, pp. 196–208.

[2]. Banerjee, A., "On a quantity discount pricing model to increase vendor profit." *Management Science vol. 32*, 1986a, pp. 1513–1517.

[3]. Banerjee, A., "A joint economic lot-size model for purchaser and vendor." *Decision Sciences vol. 17*, 1986b, pp. 292–311.

[4]. Banerjee, A., Banerjee, S., "A coordinated order-up-to inventory control policy for a single supplier and multiple buyers using electronic data interchange. " in *International Journal of Production Economics 35*, 1986, pp. 85–91.

[5]. Ben-Daya, M., Hariga, M., "Integrated single vendor single buyer model with stochastic demand and variable lead-time." in *International Journal of Production Economics 92,* 2004, pp. 75–80.

[6]. Ben-Daya, M., Hariga M, "Consignment and vendor managed inventory in single-vendor multiple buyers supply chains." *In International Journal of Production Research. Volumen 51, Issue 5, 2013.*

[7]. Boyaci, T., Gallego, G., 2002. "Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers." in *International Journal of Production Economics vol. 77 (ed. 2), 2002, pp. 95–111.*

[8]. Braglia, M., Zavanella, L., "Modelling an industrial strategy for inventory management in supply chains: The 'Consignment Stock' case." in *International Journal of Production Research* vol. 41, 2003, pp. 3793–3808.

[9]. Goyal, S.K., " An integrated inventory model for a single supplier– single customer problem." *International Journal of Production Research* vol. 15 (1), 1976, pp. 107–111.

[10]. Goyal, S.K., "A joint economic-lot-size model for purchaser and vendor: A comment." *Decision Sciences vol. 19*, 1988, pp. 236–241.

[11]. Goyal, S.K., "A one-vendor multi-buyer integrated inventory model: A comment" *European Journal of Operational Research* vol. 82, 1995, pp. 209-210.

[12]. Goyal, S.K., "On improving the single-vendor single buyer integrated production inventory model with a generalized policy." *in European Journal of Operational Research* vol. 125, 2000, pp. 429–430.

[13]. Goyal, S.K., Gupta, Y.P., "Integrated inventory models: The buyer–vendor coordination." in *European Journal of Operational Research* vol. 41, 1989, pp. 261–269.

[14]. Hill, R.M., "The single-vendor single-buyer integrated production- inventory model with a generalized policy." in *European Journal of Operational Research* vol. 97, 2001, pp. 493–499.

[15]. Hoque, M., Goyal, S.K., 2006. "A heuristic solution procedure for an integrated inventory system under controllable lead-time, with equal or unequal sized batch

shipments, between a vendor and a buyer." *International Journal of Production Economics* vol. 102 (ed. 2), 2006, pp. 217–225.

[16]. Hoque M.A., "Synchronization in the single-manufacturer multi-buyer integrated inventory supply chain," in *European Journal of Operational Research 188,* 2008, pp. 811–825.

[17]. Hoque M.A., "An alternative optimal solution technique for a single-vendor single-buyer integrated production inventory model," *International Journal of Production Research* Vol. 47, No 15. 2006

[18]. Hoque M.A.," Generalized single-vendor multi-buyer integrated inventory supply chain models with a better synchronization*,"* *Int. J. Production Economics 131* in 2011, pp. 463–472.

[19]. Hoque M.A.," A note on 'Generalized single-vendor multi-buyer integrated inventory supply chain models with a better synchronization'," in *Int. J. Production Economics.*

[20]. Hariga M, Gumus M, Ben-Daya M and Hassini, "Scheduling and lot sizing models for the single-vendor multi-buyer problem under consignment stock partnership." Journal of the Operational Research Society vol. 64, 2013, pp. 995–1009.

[21]. Hariga M, Gumus M, Hassini E, and Ben-Daya M, "A note on generalized single-vendor multi-buyer integrated inventory supply chain models with better synchronization." in *Int. J. Production Economics*. 2013

[22]. Joglekar, P.N., "Comments on a quantity discount pricing model to increase vendor profits." *Management Science 34,* 1988, pp. 1391–1398.

[23]. Joglekar, P.N., Tharthare, S., "The individually responsible and rational decision approach to economic lot sizes for one vendor and many purchasers." *Decision Sciences 21 (3*), 1990, pp. 492–506.

[24]. Kim, T., Hong, Y., Chang, S.Y., "Joint economic procurement– production–delivery policy for multiple items in a single-manufacturer, multiple-retailer system." in *International Journal of Production Economics* 103, 2006, pp. 199–208.

[25]. Lal, R., Staelin, R., " An approach for developing an optimal discount pricing policy." *Management Science 30, 1984*, pp. 1524–1539.

[26]. Lu, L., "A one-vendor multi-buyer integrated inventory model." in *European Journal of Operational Research* 81, 1995, pp. 312–323.

[27]. Valentini, G., Zavanella, L., 2003. "The Consignment Stock of inventories: Industrial case study and performance analysis." in *International Journal of Production Economics, 2003, pp.* 81–82, 215–224.

[28]. Viswanathan, S., Piplani, R., " Coordinating supply chain inventories through common replenishment epochs." *European Journal of Operational Research* vol. 129, 2001, pp. 277–286.

[29]. Woo, Y.Y., Hsu, S.L., Wu, S., "An integrated inventory model for a single vendor and multiple buyers with ordering cost reduction." in *International Journal of Production Economics vol.* 73, 2001, pp. 203–215.

[30]. Zanoni, S., Grubbstro, R.W., " A note on an industrial strategy for stock management in supply chains: Modelling and performance evaluation." *International Journal of Production Research* 42, 2001, pp. 4421–4426.

[31]. Zhou, Y.W., Wang, S.D., "Optimal production and shipment models for a single-vendor–single-buyer integrated system." in *European Journal of Operational Research* 180 (1), 2007, pp. 309–328.

[32]. Zavanella L, Zanoni S, 2009. "A one-vendor multi-buyer integrated production-inventory model: The 'Consignment Stock' case*." Int. J. Production Economics* 118,2009, pp. 225–232.

[33]. Zavanella L, Zanoni S, "A one-vendor multi-buyer integrated production-inventory model: The 'Consignment Stock' case''" *[International Journal of Production Economics, 118 (2009) 225–232]. Int. J. Production Economics 125*, 2010, pp. 212–213.

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